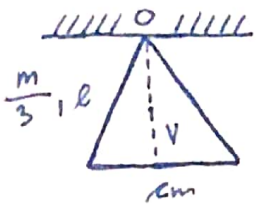
# 1. KZ - F2 2019.

**1** $M = 0{,}3\ kg$

$\ell = 5\ cm$

$T = ?$




$D = \frac{V}{2}$

$V = \frac{\sqrt{3}}{2}\ell \Rightarrow D = \frac{\sqrt{3}\ell}{4}$ (*) (1)

$D = 0{,}0216\ m$

$T = 2\pi\sqrt{\frac{I}{MgD}}$ (1)

$I = 2 \cdot I_{kraj} + I_{\circ} + md^2$, $d = V$ (2) (1)

$I_{kraj} = \frac{1}{3}m\ell^2$

$I_{\circ} = \frac{1}{12}m\ell^2$ (3) (0,5)

(3) → (2) i $m = \frac{M}{3}$

$I = 2 \cdot \frac{1}{3} \cdot \frac{M}{3}\ell^2 + \frac{1}{12} \cdot \frac{M}{3}\ell^2 + \frac{M}{3} \cdot \left(\frac{\sqrt{3}}{2}\ell\right)^2$

$I = \frac{2}{9}M\ell^2 + \frac{1}{33}M\ell^2 + \frac{1}{4}M\ell^2 = \frac{199}{396}M\ell^2$ (4)

$I = \frac{199}{396} \cdot 0{,}3 \cdot (5 \cdot 10^{-2})^2 = 3{,}769 \cdot 10^{-4}\ kgm^2$ (1)

(4), (*) → (1)

$T = 2\pi\sqrt{\frac{\frac{199}{396}M\ell^2}{M \cdot g \cdot \frac{\sqrt{3}\ell}{4}}}$

$T = 2\pi\sqrt{\frac{\frac{199}{396}\ell}{\frac{\sqrt{3}}{4}g}} = 2\pi\sqrt{\frac{\frac{199}{396} \cdot (5 \cdot 10^{-2})}{\frac{\sqrt{3}}{2} \cdot 9{,}81}}$

$T = 0{,}342\ s$ (1)

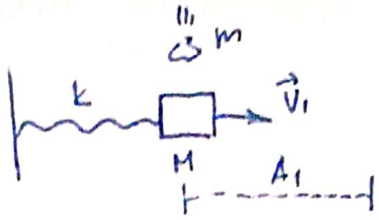
$f = \frac{1}{T} = \boxed{2{,}93\ Hz}$ (0,5)

**2** $M = 1{,}5\ kg$

$k = 0{,}12\ N/m$

$m = 15 \cdot 10^{-3}\ kg$

$A_{max} = ?$




Z.O.E. PRIJE:

$$E_1 = \frac{1}{2} k A_1^2 = \frac{1}{2} M V_1^2 \Rightarrow V_1 = \sqrt{\frac{k}{M}}\, A_1 \quad (1) \quad (1)$$

Z.O.K.G.:

$$M V_1 + 0 = (M+m) V_2 \Rightarrow V_2 = \frac{M}{M+m} \cdot V_1 \quad (2) \quad (2)$$

Z.O.E. NAKON:

$$E_2 = \frac{1}{2} (M+m) V_2^2 = \frac{1}{2} \frac{M^2}{M+m} V_1^2 = \frac{M}{M+m} \left(\frac{1}{2} M V_1^2\right) = \frac{M}{M+m} E_1 \quad (+) \quad (1)$$

$$E_2 = \frac{1}{2} k A_2^2$$

$$\Rightarrow \quad \frac{1}{2} k A_2^2 = \frac{M}{M+m} \cdot \frac{1}{2} k A_1^2$$

$$A_2 = A_1 \sqrt{\frac{M}{M+m}} \quad (1)$$

$$\boxed{A_2 = 0{,}995\, A_1} \quad (1)$$

3

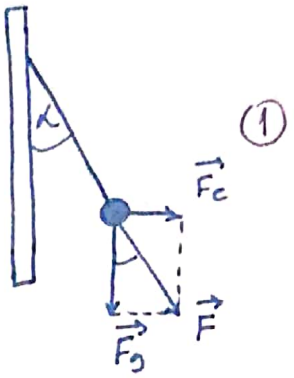
$E = \frac{\sigma}{2\varepsilon_0}$ (*) $\qquad \varepsilon_0 = 8{,}85 \cdot 10^{-12}\ C^2/Nm^2$

$\sigma = 3 \cdot 10^{-5}\ C/m^2$

$m = 10^{-3}\ kg$

$\alpha = 30°$

$Q = ?$




(1)

$tg\alpha = \frac{F_e}{F_g}$ (1) (1)

$F_e = E \cdot Q = \frac{\sigma}{2\varepsilon_0} \cdot Q$ (2) (2)

$F_g = m \cdot g$

(2) → (1)

$tg\alpha = \frac{\sigma Q}{2\varepsilon_0 mg} \Rightarrow Q = \frac{2\varepsilon_0 mg\, tg\alpha}{\sigma} = \boxed{3{,}3\ nC}$ (1)

4

$Q_\alpha = 2e \qquad e = 1{,}62 \cdot 10^{-19} C$

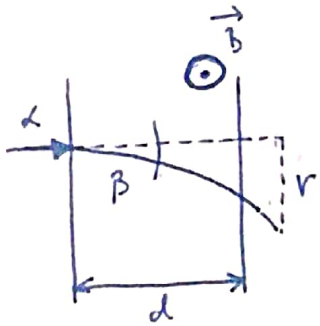
$m_\alpha = 4m_p \qquad m_p = 1{,}67 \cdot 10^{-27}\ kg$

$U = 250 \cdot 10^3\ V$

$B = 0{,}51\ T$

$d = 0{,}1\ m$

$B = ?$




$tg\beta = \frac{d}{r}$ (*) (1)

$W = E_k$

$Q_\alpha \cdot U = \frac{1}{2} m_\alpha v^2$ (1) (0,5)

$2eU = \frac{1}{2} \cdot 4m_p v^2 \Rightarrow eU = m_p v^2 \Rightarrow v = \sqrt{\frac{eU}{m_p}}$ (1) $\qquad v = 4{,}92 \cdot 10^6\ \frac{m}{s}$

$F_m = Q_\alpha v B$

$F_{cp} = m_\alpha \cdot a,\ a = \frac{v^2}{r}$

$F_m = F_{cp} \qquad \frac{m_\alpha v^2}{r} = Q_\alpha v B$ (2) (1)

(1) → (2)

$\frac{4m_p v^2}{r} = 2evB \Rightarrow r = \frac{2m_p v}{Be} = \frac{2m_p}{Be} \cdot \sqrt{\frac{eU}{m_p}} = \frac{2}{B}\sqrt{\frac{m_p U}{e}}$ (3) (0,5)

$r = 0{,}199\ m$

(3) → (*)

$$tg\beta = \frac{Bd}{2}\sqrt{\frac{e}{m_p U}}$$

$tg\beta = 0{,}150251 \Rightarrow \boxed{\beta = 8{,}5^\circ 9'}$ (1)