NEKOLIKO ZADATAKA S ISPITNIH ROKOVA

RIJEŠITI DIFERENCIJALNU JEDNADŽBU, UVRSTITI RJEŠENJE I PROVJERITI ZADOVOLJAVANJE JEDNAKOSTI:

$$x^2 + x^2 y + y^2 y' = 0$$

Rješenje

$$y^2 y' = -x^2 - x^2 y$$

$$y' = -\frac{x^2}{y^2} - \frac{x^2}{y} \quad \text{NIJE HOMOGENA}$$

$$y' = -x^2\left(\frac{1}{y^2} + \frac{1}{y}\right) \quad \text{SEPARIRANA}$$

$$y' = -x^2\left(\frac{y+1}{y^2}\right)$$

$$\int \frac{y^2\,dy}{y+1} = -\int x^2\,dx$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

$$\int \frac{y^2 dy}{y+1} = \left\{ \begin{array}{l} y^2 : (y+1) = y - 1 + \frac{1}{y+1} \\ \underline{-(y^2+y)} \\ -y \\ \underline{-(-y-1)} \\ 1 \end{array} \right\}$$

$$= \int \left(y - 1 + \frac{1}{y+1}\right) dy = \frac{y^2}{2} - y + \ln|y+1| + C$$

$$\frac{y^2}{2} - y + \ln|y+1| = -\frac{x^3}{3} + C$$

OVO JE IMPLICITNO RJEŠENJE POLAZNE (ODJ) I STOGA NIJE MOGUĆE JEDNOSTAVNO DERIVIRATI y I UVRSTITI U POLAZNU JEDNADŽBU RADI PROVJERE

RIJEŠITI DIFERENCIJALNU JEDNADŽBU $x^2 + yy' = 1$ UZ UVJET $y(0) = 1$

Rješenje

$$y y' = 1 - x^2$$

$$y' = (1 - x^2) \cdot \frac{1}{y} \quad \text{SEPARIRANA (ODJ)}$$

$$\int y\,dy = \int (1 - x^2)\,dx$$

$$\int y\,dy = \frac{y^2}{2} + C$$

$$\int (1 - x^2)\,dx = x - \frac{x^3}{3} + C$$

$$\frac{y^2}{2} = x - \frac{x^3}{3} + C \quad \text{IMPLICITNO RJEŠENJE}$$

$$y(x) = \pm\sqrt{2x - \frac{2}{3}x^3 + C} \quad \text{EKSPLICITNO RJEŠENJE}$$

RIJEŠITI DIFERENCIJALNU JEDNADŽBU: $y'+3y+4x=2$

Rješenje

$y'+3y=2-4x$ LINEARNA ODJ

1. korak

$$y'+3y=0$$
$$y'=-3y$$
$$\int\frac{dy}{y}=-\int 3\,dx$$
$$\ln|y|=-3x+C$$
$$y(x)=Ce^{-3x}$$

2. korak

$$C\mapsto C(x)$$
$$y(x)=C(x)\cdot e^{-3x}$$
$$y'(x)=C'(x)\cdot e^{-3x}+C(x)\cdot(-3)\cdot e^{-3x}$$

UVRSTITI U POLAZNU ODJ

$$C'(x)\cdot e^{-3x}\underbrace{-3C(x)e^{-3x}+3C(x)e^{-3x}}_{=0}=2-4x$$
$$C'(x)\cdot e^{-3x}=2-4x$$
$$C'(x)=e^{3x}(2-4x)$$
$$C(x)=\int e^{3x}(2-4x)dx$$
$$=2\int e^{3x}dx-4\underbrace{\int xe^{3x}dx}_{\text{PARCIJALNA INTEGRACIJA}}$$
$$\int xe^{3x}dx=\frac{1}{9}e^{3x}(3x-1)$$
$$=-\frac{2}{9}e^{3x}(6x-5)+C$$
$$\Rightarrow y(x)=\left[-\frac{2}{9}e^{3x}(6x-5)+C\right]\cdot e^{-3x}$$
$$y(x)=-\frac{2}{9}(6x-5)+Ce^{-3x}$$

RIJEŠITI DIFERENCIJALNU JEDNADŽBU: $y''+2y'=2x$

Rješenje

OVO JE NEHOMOGENA ODJ 2. REDA S KONSTANTNIM KOEFICIJENTIMA

1. korak

$$y''+2y'=0$$
$$\lambda^2+2\lambda=0\Rightarrow\begin{cases}\lambda_1=0\\ \lambda_2=-2\end{cases}$$
$$y_H(x)=C_1e^{0x}+C_2e^{-2x}$$
$$=C_1+C_2e^{-2x}$$

2. korak

zbog $f=2x=e^{0x}P_1(x)\Rightarrow\begin{cases}a=0\\ m=1\\ r=1\end{cases}$

$$\Rightarrow Y(x)=xe^{0x}(Ax+B)=Ax^2+Bx$$
$$Y'(x)=\ldots=2Ax+B$$
$$Y''(x)=2A$$

UVRSTITI U POLAZNU ODJ

$$Y''+2Y'=2x$$
$$2A+2(2Ax+B)=2x$$
$$2A+4Ax+2B=2x$$

rastav na 2 jednadžbe

uz x: $4A=2\Rightarrow A=\frac{1}{2}$

bez x: $2A+2B=0\Rightarrow B=-\frac{1}{2}$

$$\Rightarrow Y(x)=\frac{1}{2}x^2-\frac{1}{2}x$$

OPĆE RJEŠENJE POLAZNE ODJ:

$$y(x)=C_1+C_2e^{-2x}+\frac{1}{2}x^2-\frac{1}{2}x$$