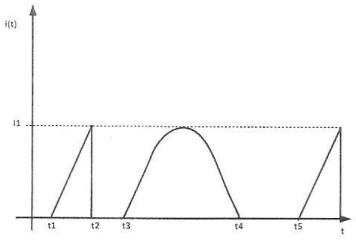
ogledni primjerak

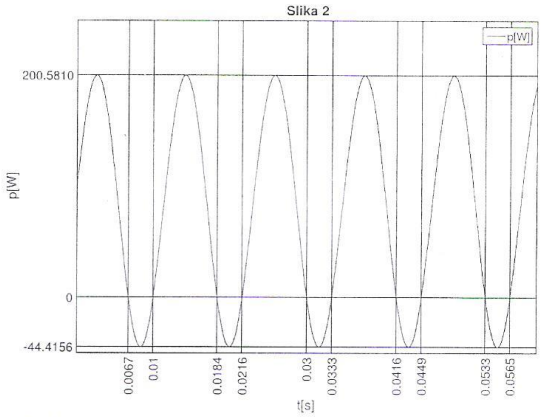
1.kolokvij AV, OET 2, preddiplomski studij elektrotehnike, 18.04.2016. Ime i prezime:

1.ZADATAK: Kroz otpornik otpora R protječe struja valnog oblika (sastavljen od trokutastog i sinusnog valnog oblika). prikazanog na slici 1. Odredite kolika se količina topline razvije na otporniku u jednom periodu valnog obika struje. Zadano: R= 5 Ohm, t1= 1 s, t2= 2 s, t3= 3 s, t4= 5 s, t5= 9 s, I1=2 A. (**15 bodova**)



Slika 1.

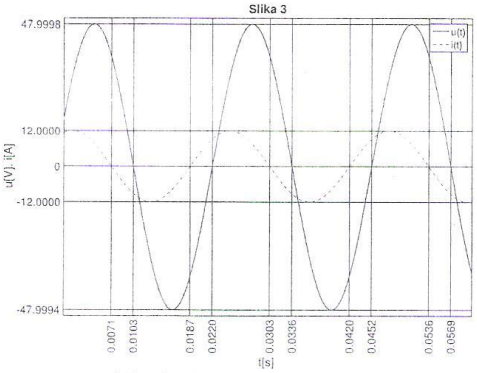
Slika 2

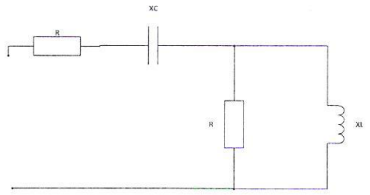


2.ZADATAK: Valni oblik trenutne snage na nekom trošilu prikazan je na slici 2. Odredite djelatnu, jalovu i prividnu snagu toga trošila. Pmax+ = 200,58 W Pmax-= -44,42 W (**10 bodova**)

3.ZADATAK: Na nekom trošilu snimljeni su valni oblici struje i napona prikazani na slici 3. Odredite fazore napona i struje te impedanciju ovog trošila ako je frekvencija 123 Hz. (**10 bodova**)

Slika 3



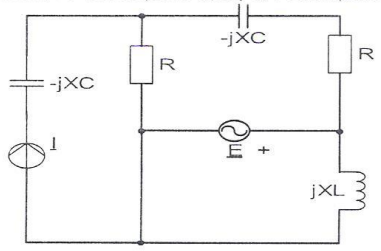


Slika 4.

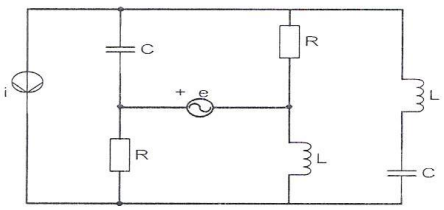
4.ZADATAK: Električni krug na slici 4 je u rezonanciji. Odredite XL ako je zadano R= 5 Ohm i XC= 6,9 Ohm. (**20 bodova**)

5.ZADATAK: Za strujni krug prikazan na slici 5 metodom izravne primjene Kirchhoffovih zakona odredite struje svih grana u mreži. Zadano: $\underline{E}$=231+j155 V, $\underline{I} = 20e^{j121^\circ}$ A, R=74 Ω, C= 6 μF, L=50 mH (**20 bodova**)

6.ZADATAK: Odredite kompleksnu snagu strujnog izvora u mreži na slici 6 primjenom metode konturnih struja. Zadano: e=110sin(800t+0,2π) V, i=17sin(800t-1,2π) A, R=74 Ω, C= 7 μF, L=60 mH.(**25 bodova**)



SLIKA 5



SLIKA 6

1. KOLOKVIJ - OET2 AV

- RJEŠENJA -

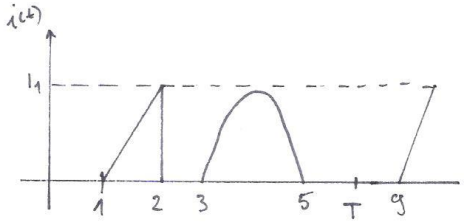
Zadatak 1.

$R = 5\,\Omega$

$i(t) \rightarrow$ slika

$W[J] = ?$; 1 period

$I_1 = 2\,A$



$I_{ef\Delta} = \frac{I_1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = 1{,}15\,A$; $I_{ef\wedge} = \frac{I_1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1{,}41\,A$

$T = t_5 - t_1 = 8\,s$; $\tau_\Delta = t_2 - t_1 = 2 - 1 = 1\,s$; $\tau_\wedge = t_4 - t_3 = 5 - 3 = 2\,s$

$I_{ef\Delta 1} = I_{ef\Delta}\sqrt{\frac{\tau_\Delta}{T}} = 1{,}15\sqrt{\frac{1}{8}} = 0{,}41\,A$; $I_{ef\wedge 1} = I_{ef\wedge}\sqrt{\frac{\tau_\wedge}{T}} = 1{,}41\sqrt{\frac{2}{8}} = 0{,}71\,A$

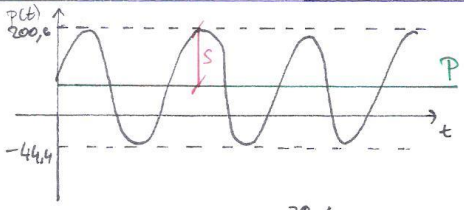
$I_{ef} = \sqrt{I_{ef\Delta 1}^2 + I_{ef\wedge 1}^2} = \sqrt{0{,}41^2 + 0{,}71^2} = 0{,}82\,A$; $W = P \cdot T = I_{ef}^2 \cdot R \cdot T = 0{,}82^2 \cdot 5 \cdot 8 = 26{,}89\,J$

Zadatak 2.

$P_{max} = 200{,}58\,W$

$P_{min} = -44{,}42\,W$

$P; Q; S = ?$



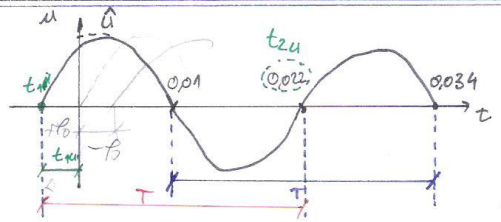
$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{P_{max} - |P_{min}|}{2} = \frac{200{,}6 - |-44{,}4|}{2} = 78{,}1\,W$; $S = P_{max} - P = 200{,}58 - 78{,}1 = 122{,}48\,VA$

$Q = \sqrt{S^2 - P^2} = \sqrt{122{,}48^2 - 78{,}1^2} = 94{,}35\,VAr$

Zadatak 3.

$U = 48\,V$

$I = 12\,A$



$T = 0{,}034 - 0{,}01 = 0{,}024\,s$

$f = \frac{1}{T} = \frac{1}{0{,}024} = 41{,}67\,Hz \Rightarrow \omega = 2\pi f = 261{,}8\,s^{-1}$

$\underline{U} = \frac{\hat{U}}{\sqrt{2}} e^{j\varphi_u} = 33{,}94\, e^{+j0{,}17\pi}\,V$

$\underline{I} = \frac{\hat{I}}{\sqrt{2}} e^{j\varphi_i} = 8{,}49\, e^{+j0{,}44\pi}\,A$

$t_{1u} = t_{2u} - T = 0{,}022 - 0{,}024 = -0{,}002\,s$

$\varphi_u = \omega t_{1u} = 261{,}8 \cdot (-0{,}002) = -0{,}52 = -0{,}17\pi$

$t_{1i} = t_{2i} - T = 0{,}0187 - 0{,}024 = -0{,}0053\,s$

$\varphi_i = \omega \cdot t_{1i} = 261{,}8 \cdot (-0{,}0053) = -1{,}39 = -0{,}44\pi$

$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{33{,}34\, e^{+j0{,}17\pi}}{8{,}49\, e^{+j0{,}44\pi}}$

$\underline{Z} = 4\, e^{-j0{,}27\pi}\,\Omega$

$u = U_m \sin(\omega t \pm \varphi_0)$

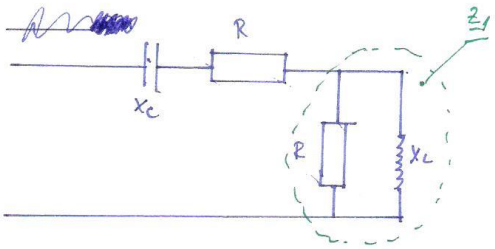
zak 4.

$R = 5\,\Omega$

$X_c = 6{,}9\,\Omega$

Rezonancija

$X_L = ?$



$$\underline{Z}_1 = \frac{R\,jX_L}{R+jX_L}\cdot\frac{R-jX_L}{R-jX_L} = \frac{R^2jX_L - R(jX_L)^2}{R^2+X_L^2} = \frac{X_L^2 R}{R^2+X_L^2} + j\frac{R^2X_L}{R^2+X_L^2}$$

$$\mathrm{Im}\{\underline{Z}_{uk}\} = 0 \Rightarrow -X_c + \frac{R^2X_L}{R^2+X_L^2} = 0 \;/\;(R^2+X_L^2)$$

$$\underbrace{-X_c}_{c}R^2 + \underbrace{X_c}_{a}X_L^2 + \underbrace{R^2}_{b}X_L = 0$$

$$X_{L1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$X_{L1,2} = \frac{-R^2 \pm \sqrt{(R^2)^2 - 4(-X_c)^2\cdot R^2}}{-2X_c}$$

$2^4 < 4X_c^2R^2 \Rightarrow$ kompleksno rješenje!!!

Zadatak 6.

$\underline{E} = 231 + j155\,V = 278{,}18\,e^{j0{,}59}\,V$

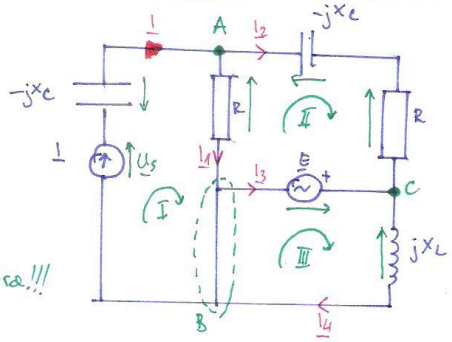
$\underline{I} = 20\,e^{j2{,}11}\,A = -10{,}27 + j17{,}16\,A$

$R = 74\,\Omega \qquad X_c = \frac{1}{\omega C} = 530{,}52\,\Omega$

$C = 6\,\mu F$

$L = 50\,mH \qquad X_L = \omega L = 15{,}71\,\Omega$

zbog uredności izostavljam oznake fazora!!!



I. $U_s - I(-jX_c) - I_1(R) = 0$

II. $-E + I_1R - I_2(-jX_c) - I_2(R) = 0$

III. $E - I_4 jX_L = 0$

A. $I - I_1 - I_2 = 0$

B $I_1 + I_4 - I_3 - I = 0$

C $I_2 + I_3 - I_4 = 0$

iz III. $E = I_4 jX_L$; $I_4 = \frac{E}{jX_L} = \frac{278\,e^{j0{,}59}}{j\,15{,}71}$

$I_4 = 17{,}69\,e^{j0{,}98} = 9{,}85 - j14{,}69\,A$

iz A i II!

$-\underline{E} + (\underline{I} - \underline{I}_2)R + \underline{I}_2 jX_c - \underline{I}_2 R = 0$

$-E + I\cdot R - I_2R + I_2jX_c - I_2R = 0$

$I_2(-R - R + jX_c) = E - I\cdot R$

$$I_2 = \frac{E - I\cdot R}{-2R + jX_c} = \frac{231 + j155 - 74(-10{,}27 + j17{,}16)}{-2\cdot 74 + j\,530{,}52}$$

$$I_2 = \frac{990{,}98 - j1114{,}84}{-148 + j530{,}52} = 2{,}71\,e^{-j2{,}68} = -2{,}43 - j1{,}21\,A$$

$I_1 = I - I_2 = -10{,}27 + j17{,}16 + 2{,}43 + j1{,}21 =$

$= -7{,}84 + j18{,}37 = 19{,}97\,e^{j1{,}97}\,A$

$I_3 = I_4 - I_2 = 9{,}85 - j14{,}69 + 2{,}43 + j1{,}21 =$

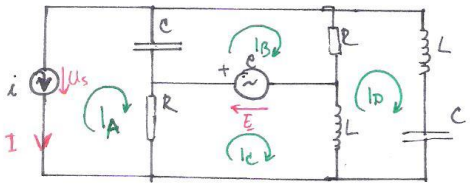
$= 12{,}28 - j15{,}9 = 20{,}09\,e^{-j0{,}91}\,A$

Zk 6.

$110 \sin(800t + 0{,}2\pi) \Rightarrow \underline{E} = \frac{110}{\sqrt{2}} e^{j0{,}2\pi} = 77{,}78\, e^{j0{,}2\pi}\ V$

$i = 17 \sin(800t - 1{,}2\pi)$ $\quad \underline{I} = \frac{17}{\sqrt{2}} e^{-j1{,}2\pi} = 12{,}02\, e^{-j1{,}2\pi}\ A$

$R = 7\Omega$; $C = 7\mu F$; $L = 60mH$



$\omega = 800$

$X_C = \frac{1}{\omega C} = \frac{1}{800 \cdot 7 \cdot 10^{-6}} = 178{,}57\ \Omega$

$X_L = \omega L = 800 \cdot 60 \cdot 10^{-3} = 48\ \Omega$

$I_A(R - jX_C) - I_B(-jX_C) - I_C R = -\hat{U}_S$ (tražiti se!)

$I_B(R - jX_C) - I_A(-jX_C) - I_D(R) = E$

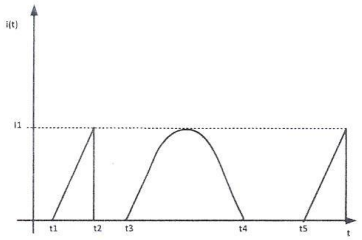
$I_C(R + jX_L) - I_A R - I_D(jX_L) = -E$

$I_D(R + 2jX_L - jX_C) - I_B(R) - I_C(jX_L) = 0$

$I_A = -1$

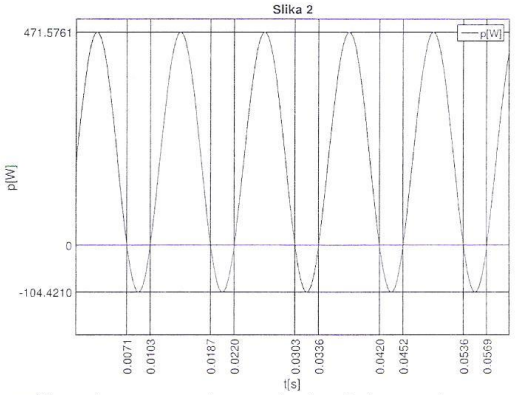
1.kolokvij AV, OET 2, preddiplomski studij elektrotehnike, 18.04.2016. Ime i prezime:

1.ZADATAK;: Kroz otpornik otpora R protječe struja valnog oblika prikazanog na slici 1 (sastavljen od trokutastog i sinusnog valnog oblika). Odredite kolika se količina topline razvije na otporniku u jednom periodu valnog oblika struje. Zadano: R= 5 Ohm, t1= 1 s, t2= 2 s, t3= 3 s, t4= 7 s, t5= 9 s, I1=2 A. (**15 bodova**)



Slika 1.

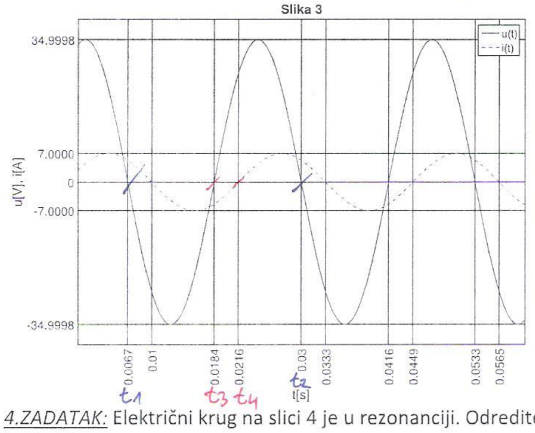
Slika 2

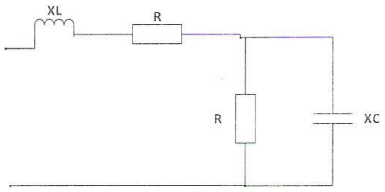


2.ZADATAK: Valni oblik trenutne snage na nekom trošilu prikazan je na slici 2. Odredite djelatnu, jalovu i prividnu snagu toga trošila. Pmax+ =471,58 W Pmax-=-104,42 W (**10 bodova**)

3.ZADATAK: Na nekom trošilu snimljeni su valni oblici struje i napona prikazani na slici 3. Odredite fazore napona i struje te impedanciju ovog trošila ako je frekvencija 71 Hz. (**10 bodova**)

Slika 3



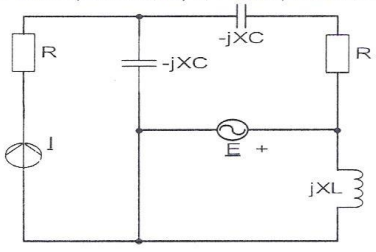


Slika 4.

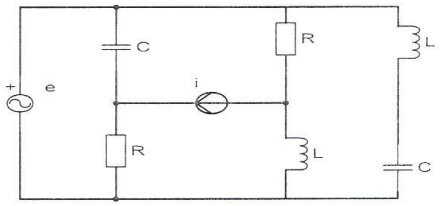
4.ZADATAK: Električni krug na slici 4 je u rezonanciji. Odredite XC ako je zadano R= 10 Ohm i XL= 10 Ohm. (**20 bodova**)

5.ZADATAK: Za strujni krug prikazan na slici 5 metodom izravne primjene Kirchhoffovih zakona odredite struje svih grana u mreži. Zadano: $\underline{E}$=200+j175 V, $\underline{I} = 20e^{-j57^\circ}$ A, R=74 Ω, C= 6 μF, L=50 mH (**20 bodova**)

6.ZADATAK: Odredite kompleksnu snagu strujnog izvora u mreži na slici 6 primjenom metode konturnih struja. Zadano: e=220sin(1000t+0,2π) V, i=13sin(1000t-1,2π) A, R=74 Ω, C= 6 μF, L=50 mH.(**25 bodova**)



SLIKA 5



SLIKA 6

1. KOLOKVIJ OET2 -AV

- RJEŠENJA -

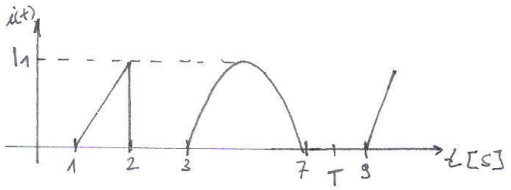
Zadatak 1.

$R = 5\,\Omega$

$i(t) \rightarrow$ slika

$W[J] = ?$; 1 period

$I_1 = 2\,A$



$$I_{ef\Delta} = \frac{I_1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = 1{,}15\,A \quad ; \quad I_{ef\wedge} = \frac{I_1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1{,}41\,A$$

$$T = t_5 - t_1 = 9 - 1 = 8\,s \quad ; \quad \tau_\Delta = t_2 - t_1 = 2 - 1 = 1\,s \quad ; \quad \tau_\wedge = t_4 - t_3 = 7 - 3 = 4\,s$$

$$I_{ef\Delta 1} = I_{ef\Delta}\sqrt{\frac{\tau_\Delta}{T}} = 1{,}15\sqrt{\frac{1}{8}} = 0{,}41\,A \quad ; \quad I_{ef\wedge 1} = I_{ef\wedge}\sqrt{\frac{\tau_\wedge}{T}} = 1{,}41\sqrt{\frac{4}{8}} = 1\,A$$

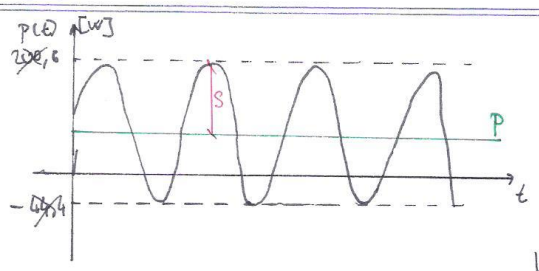
$$I_{ef} = \sqrt{I_{ef\Delta 1}^2 + I_{ef\wedge 1}^2} = \sqrt{0{,}41^2 + 1^2} = 1{,}08\,A \quad ; \quad W = P \cdot T = I_{ef}^2 \cdot R \cdot T = 1{,}08^2 \cdot 5 \cdot 8 = 43{,}23\,[Ws]\,[J]$$

Zadatak 2.

$P_{max} = \cancel{200{,}58\,W} = 471{,}58\,W$

$P_{min} = \cancel{-44{,}42\,W} = -104{,}42$

$P = ?$ $Q = ?$ $S = ?$



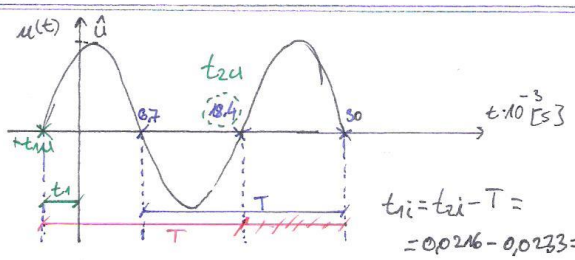
$$P = \frac{1}{T}\int_0^T p(t)\,dt = \text{srednja vrijednost sinusoide!} = \frac{P_{max} - |P_{min}|}{2} = \frac{\cancel{200{,}58} - \cancel{44{,}42}}{2} = \quad \text{kriva grupa}$$

$$P = \frac{P_{max} - |P_{min}|}{2} = \frac{471{,}58 - 104{,}42}{2} = 183{,}58\,W \quad ; \quad S = P_{max} - P = 471{,}58 - 183{,}58 = 288\,VA \quad ; \quad Q = \sqrt{S^2 - P^2} = 221{,}91\,VAr$$

Zadatak 3.

$U = 35\,V$

$\uparrow = 7\,A$



$$T = (30 - 6{,}7)\cdot 10^{-3} = 0{,}0233\,s$$

$$f = \frac{1}{T} = 42{,}91\,Hz \quad ; \quad \omega = 2\pi f = 2\pi \cdot 42{,}91 = 269{,}61\,s^{-1}$$

$$\underline{U} = \frac{\hat{U}}{\sqrt{2}} e^{j\varphi_u} = 24{,}75\, e^{+j0{,}42\pi}\ V$$

$$\underline{I} = \frac{\hat{\uparrow}}{\sqrt{2}} e^{j\varphi_i} = 4{,}95\, e^{+j0{,}15\pi}\ A$$

$$t_{1i} = t_{2i} - T = 0{,}0216 - 0{,}0233 = -0{,}0017\,s$$

$$\varphi_i = \omega\, t_{1i} = 269{,}61 \cdot (-0{,}0017) = -0{,}15\,\pi$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{24{,}75\, e^{+j0{,}42\pi}}{4{,}95\, e^{j0{,}15\pi}} = 5\, e^{+j0{,}27\pi}\ \Omega$$

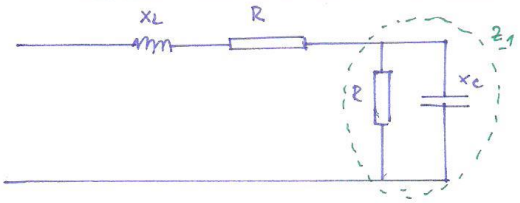
fazni pomak!:

$$t_{1u} = t_{2u} - T = 0{,}0184 - 0{,}0233$$

$$t_{1u} = -0{,}0049\,s$$

$$\varphi_u = \omega \cdot t_{1u} = 269{,}61 \cdot (-0{,}0049)$$

$$\varphi_u = \angle 0{,}42\,\pi$$



Ω

?

rezonancija

$$\underline{Z}_1 = \frac{-Rjx_c}{R-jx_c}\cdot\frac{R+jx_c}{R+jx_c} = \frac{x_c^2R - jR^2x_c}{R^2+x_c^2} = \frac{x_c^2R}{R^2+x_c^2} - j\frac{R^2x_c}{x_c^2+R^2}$$

$$\mathrm{Im}\{\underline{Z}_{uk}\} = 0 \Rightarrow \quad X_L - \frac{R^2x_c}{R^2+x_c^2} = 0 \;/\cdot(R^2+x_c^2)$$

$$\underbrace{R^2}_{c}X_L + \underbrace{X_L}_{a}x_c^2 \underbrace{- R^2}_{b}x_c = 0 \qquad x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x_{c1,2} = \frac{R^2 \pm \sqrt{(-R^2)^2 - 4\cdot R^2\cdot X_L^2}}{2X_L} \;;\quad R^4 < 4R^2X_L^2 \Rightarrow \text{Kompleksno rješenje}$$

$$x_{c1,2} \in \mathbb{C}$$

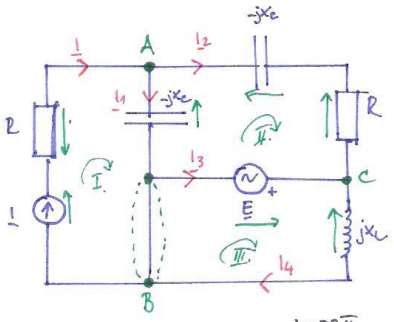
Zadatak 6.

$$\underline{E} = 200 + j175 = 265{,}75\, e^{j0{,}23\pi}\ V$$

$$\underline{I} = 20e^{-j57^\circ}\ A = 20\, e^{-j0{,}32\pi}\ A = 10{,}71 - j16{,}89\ A$$

$$R = 74\,\Omega \;;\; C = 6\,\mu F \;;\; L = 50\,mH$$

$$X_c = \frac{1}{\omega C} = 530{,}52\,\Omega \qquad X_L = \omega L = 15{,}71\,\Omega$$



zbog urednosti izostavljam oznake fazora

I. $+U_s - IR - I_1(-jx_c) = 0$

II. $-E + I_4(-jx_c) - I_2(-jx_c) - I_2R = 0$

III. $+E - I_4 jX_L = 0$

A $I - I_1 - I_2 = 0 \Rightarrow I_1 = I - I_2$

B $I_1 + I_4 - I - I_3 = 0$

C $I_2 + I_3 - I_4 = 0$

iz III. $E = I_4 jX_L$; $I_4 = \frac{E}{jX_L} = \frac{265{,}75\, e^{j0{,}25\pi}}{j15{,}71}$

$I_4 = 16{,}92\, e^{-j0{,}27\pi}\ A = 11{,}18 - j12{,}69\ A$

iz A u II!

$$-E + (I - I_2)(-jx_c) - I_2(-jx_c) - I_2R = 0$$

$$-E - jx_cI + jx_cI_2 + jx_cI_2 - I_2R = 0$$

$$I_2(2jx_c - R) = E + jx_cI$$

$$I_2 = \frac{E + jx_cI}{2jx_c - R}$$

$$I_2' = \frac{200 + j175 + j530{,}52\cdot 20\, e^{-j0{,}32\pi}}{j1061{,}04 - 74}$$

$$I_2 = \frac{200 + j175 + 10610{,}4\, e^{j0{,}18\pi}}{-74 + j1061{,}04}$$

$$I_2 = 10{,}23\, e^{-j0{,}34\pi}\ A = 4{,}93 - j8{,}96\ A$$

$$I_1 = I - I_2 = 10{,}71 - j16{,}89 - 4{,}93 + j8{,}96 =$$

$$= 5{,}78 - j7{,}93 = 9{,}81\, e^{-j0{,}3\pi}\ A$$

$$I_3 = I_4 - I_2 = 11{,}18 - j12{,}69 - 4{,}93 + j8{,}96 =$$

$$= 6{,}25 - j3{,}73 = 7{,}28\, e^{-j0{,}17\pi}\ A$$

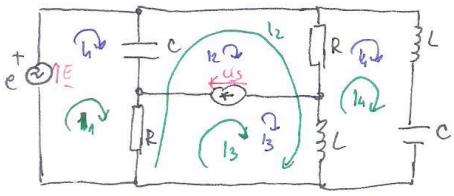
$220 \sin(1000t + 0{,}2\pi) \Rightarrow \underline{E} = \frac{220}{\sqrt{2}} e^{j0{,}2\pi}$ V

$13 \sin(1000t - 1{,}2\pi) \Rightarrow \underline{I} = \frac{13}{\sqrt{2}} e^{-j1{,}2\pi}$ A

$R = 74\,\Omega$

$C = 6\,\mu F \Rightarrow X_C = \frac{1}{\omega C} = 166{,}66\,\Omega$

$L = 50\,mH \Rightarrow X_L = \omega L = 50\,\Omega$



Preporučene jednadžbe (struja $\underline{I}$ je $\underline{I} = -\underline{I}_3$)

$$I_1(R - jX_C) - I_2(R - jX_C) - I_3(R) = E$$

$$I_2(2R - jX_C + jX_L) - I_1(R - jX_C) + I_3(R + jX_L) - I_4(R + jX_L) = 0$$

$$I_3(R + jX_L) - I_1(R) + I_2(R + jX_L) - I_4 jX_L = -U_s$$

$$I_4(R + 2jX_L - jX_C) - I_2(R + jX_L) - I_3 jX_L = 0$$

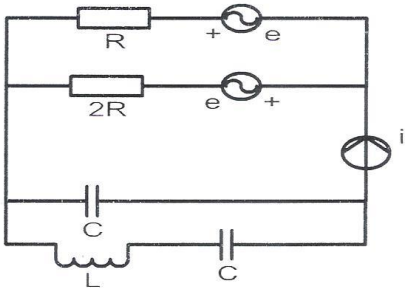
$$I_3 = -I$$

s 15 bodova se priznavalo i plave konturne struje ako su napisane jednadžbe i X_L; X_C; $\underline{E}$; $\underline{I}$

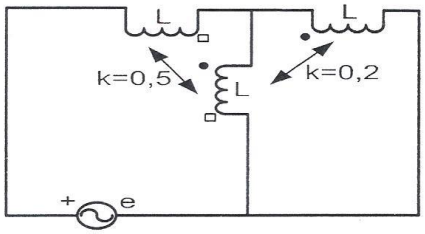
2.kolokvij AV 14.06.2016. Ogledni primjerak Ime i prezime:

OET 2, preddiplomski studij elektrotehnike

1.ZADATAK: Odredite kompleksnu snagu strujnog izvora u nadomijesnoj shemi na slici 1 primjenom metode potencijala (napona) čvorova. Poznato je: e = 200*sin(500*t+π/6) V, i = 2*sin(500*t-π/4) A, R = 10 Ω, C= 10 μF i L = 500 mH. Da li izvor troši ili daje djelatnu snagu u mrežu? **(15 bodova)** Sve do kraja ok, umm. greška (10)



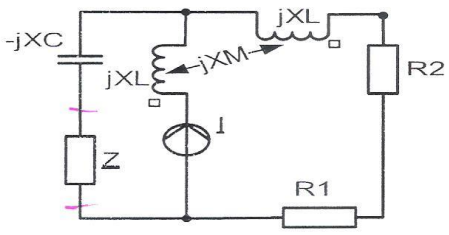
SLIKA 1



SLIKA 2

2.ZADATAK: Odredite fazor napona koji valada na krajevima srednje zavojnice na slici 2. Poznato je: frekvencija izvora 100 Hz, naponski izvor generira napon sinusnog valnog oblika amplitude od 141,42 V. Iznosi faktora magnetskih veza zadani su na shemi. **(15 bodova)**

3.ZADATAK: Izračunajte najveću djelatnu snagu koja se može razviti na impedanciji Z u nadomijesnoj shemi na slici 3. Zadano je: XC=XL=R2=R1=5 Ω, XM=2,5 Ω, I = 2 A. **(15 bodova)**



SLIKA 3

4.ZADATAK: Trofazni sustav sastoji se od trofaznog izvora u zvijezda spoju i trofaznog trošila u trokut spoju. Izračunajte kompleksnu snagu trofaznog trošila. Impedancije vodova zanemarite. Zadano je: fazni napon izvora U = 200 V, Z1= 5+j5 Ω, Z2=5+j10 Ω, Z3=5-j10 Ω. Na temelju teksta zadatka nacrtajte odgovarajuću nadomijsenu shemu. **(15 bodova)**

5.ZADATAK: Trofazno je trošilo spojeno u zvijezdu s neutralnim vodičem. Impedancije faza trošila su: Z1=10∠0°Ω , Z2=10∠-30°Ω i Z3=10∠30°Ω, a linijski napon trofaznog sustava je U = 250 V te je impedancija neutralnog vodiča Z0= 1 ∠0°Ω. Impedancije faznih vodiča trofaznog sustava zanemarite. Odredite kompleksnu snagu trošila. Na temelju teksta zadatka nacrtajte odgovarajuću nadomijsenu shemu. **(20 bodova)**

6.ZADATAK: Jednofazni transformator ima broj zavoja primarnog namota 10000 i broj zavoja sekundarnog namota 2000. Elementi nadomijsne sheme ovog transformatora imaju sljedeće vrijednosti: djelatni otpor namota primara je 0,5 Ω, djelatni otpor namota sekundara je 0,08 Ω, rasipna reaktancija primara je 1 Ω, rasipna je reaktancija sekundara 0,2 Ω, admitancija u poprečnoj grani nadomijesne sheme iznosi 0,0002-j*0,0004 S, nazivni napon primara je 6300 V. Odredite korisnost ovog transformatora i snagu trošila kada je na njegov sekundar priključeno trošilo koje ima impednaciju Zt = 1+j*0.5 Ω {**20 bodova**)

Osnove elektrotehnike 2

2. kolokvij AV - RJEŠENJA

Zadatak 1.

$e = 200 \sin(500t + \pi/6)$ V

$i = 2 \sin(500t - \pi/4)$ V

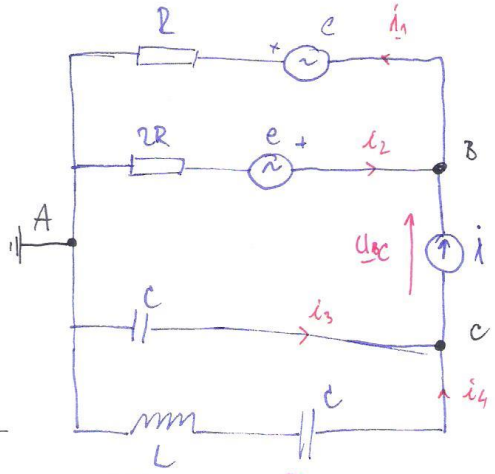
$R = 10\ \Omega$

$C = 10\ \mu F$

$L = 500$ mH

Si = ? ; Potencijali čvorova.

Izvor ili trošilo.



$\underline{E} = \frac{200}{\sqrt{2}} e^{j\pi/6} = 141{,}42\, e^{j\pi/6}$ V ; $\underline{I} = \frac{2}{\sqrt{2}} e^{-j\pi/4} = \sqrt{2}\, e^{-j\pi/4}$ A

$\omega = 500\ s^{-1}$; $X_L = \omega L = 500 \cdot 0{,}5 = 250\ \Omega$; $X_C = \frac{1}{\omega C} = \frac{1}{500 \cdot 5 \cdot 10^{-6}} = 200\ \Omega$

Metoda potencijala čvorova:

$\underline{\varphi}_A = 0$

$\underline{\varphi}_B \left(\frac{1}{R} + \frac{1}{2R} \right) = \frac{-1}{R} \underline{E} + \frac{1}{2R} \underline{E} + \underline{I}$

$\underline{\varphi}_C \left(\frac{1}{-jX_L} + \frac{1}{jX_L - jX_C} \right) = -\underline{I}$

$\underline{\varphi}_B \left(\frac{2R + R}{2R \cdot R} \right) = \underline{E} \left(\frac{1}{2R} - \frac{1}{R} \right) + \underline{I}$

$\underline{\varphi}_B \left(\frac{3}{2R} \right) = \underline{E} \left(\frac{R - 2R}{2R \cdot R} \right) + \underline{I}$

$\underline{\varphi}_B \left(\frac{3}{2R} \right) = \underline{E} \frac{-1}{2R} + \underline{I}$

$\underline{\varphi}_B\, 0{,}15 = \underline{E}\, (-0{,}05) + \underline{I}$

$$\underline{\varphi}_B = -\frac{1}{3}\underline{E} + \frac{1}{0,15}\underline{I}$$

$$\underline{\varphi}_B = -\frac{1}{3}\,200\,\frac{1}{\sqrt{2}}\,e^{j\pi/6} + \frac{1}{0,15}\,\sqrt{2}\,e^{-j\pi/4}$$

$$\underline{\varphi}_B = -47,14\,e^{j\pi/6} + 9,43\,e^{-j\pi/4}\ V$$

$$\underline{\varphi}_B = -40,85 - j23,57 + 6,67 - j6,67$$

$$\underline{\varphi}_B = -34,18 - j30,24\ V = -45,64\,e^{j0,23\pi}$$

2

$$\underline{\varphi}_C\left(j\frac{1}{X_C} - j\frac{1}{X_L - X_C}\right) = -\underline{I}$$

$$\underline{\varphi}_C\left(j\left(\frac{1}{X_C} - \frac{1}{X_L - X_C}\right)\right) = -\underline{I}$$

$$\underline{\varphi}_C\, j\,\frac{X_L - X_C - X_C}{X_L X_C - X_C^2} = -\underline{I}$$

$$\underline{\varphi}_C\, j\,\frac{X_L - 2X_C}{X_L X_C - X_C^2} = -\underline{I}$$

$$\underline{\varphi}_C = -\underline{I}\,\frac{1}{j}\,\frac{X_L X_C - X_C^2}{X_L - 2X_C}$$

$$\underline{\varphi}_C = j\,\underline{I}\,\frac{X_L X_C - X_C^2}{X_L - 2X_C} = j\sqrt{2}\,e^{-j\pi/4}\,\frac{250\cdot 200 - 200^2}{250 - 400}$$

$$\underline{\varphi}_C = -j\sqrt{2}\,66,66\,e^{-jT/4} = \sqrt{2}\,66,66\,e^{-j\pi/2}\,e^{-j\pi/4}$$

$$\underline{\varphi}_C = 94,27\,e^{-j\frac{3\pi}{4}} = -94,27\,e^{j\pi/4} = -66,66 - j66,66\ V$$

2

$$\underline{U}_{BC} = \underline{\varphi}_B - \underline{\varphi}_C = +66,66 + j66,66 - 34,18 - j30,24$$

$$\underline{U}_{BC} = 32,48 + j36,42\ V = 48,8\,e^{j0,26\pi}\ V$$

$$\underline{U}_{BC} = 48,8\,e^{j0,26T}\ V$$

3

Kompleksna snaga izvora I:

$$\underline{S}_i = \underline{U}_{BC} \cdot \underline{I}^* =$$

$$= 48{,}8\, e^{j0{,}26\pi} \cdot \sqrt{2}\, e^{j\pi/4}$$

$$= 69{,}01\, e^{j0{,}51\pi} = -2{,}17 + j\,50{,}31 \text{ VA}$$

$$= -2{,}17 + j\,68{,}98 \text{ VA} \qquad 4$$

$P_i = -2{,}17 \Rightarrow$ Izvor uzima snagu iz mreže. 2 [Z 15]

Zadatak 2.

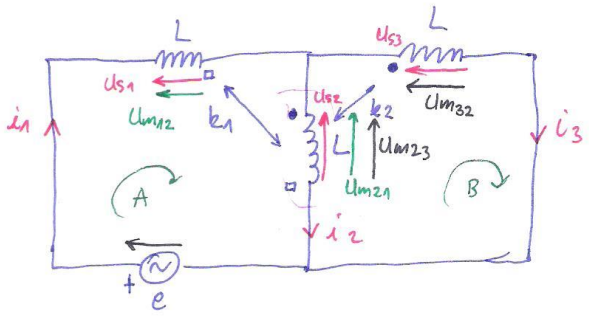
$f = 50\,\text{Hz}$

$\hat{E} = 141{,}42\,\text{V}$

$k_1 = 0{,}5$

$k_2 = 0{,}2$

$U_{L2} = ?$



$$\underline{U}_{L2} = \underline{U}_{s2} + \underline{U}_{m21} + \underline{U}_{m23}$$

KZN i KZS:

$$+\underline{E} - \underline{U}_{s1} - \underline{U}_{m12} - \underline{U}_{s2} - \underline{U}_{m21} - \underline{U}_{m23} = 0$$

$$+\underline{U}_{s2} + \underline{U}_{m21} + \underline{U}_{m23} - \underline{U}_{s3} - \underline{U}_{m32} = 0 \qquad L$$

$$\underline{I}_1 = \underline{I}_2 + \underline{I}_3$$

$$\underline{E} - \underline{I}_1 jX_L - \underline{I}_2 jX_{M1} - \underline{I}_2 jX_L - \underline{I}_1 jX_{M1} - \underline{I}_3 jX_{M2} = 0$$

$$\underline{I}_2 jX_L + \underline{I}_1 jX_{M1} + \underline{I}_3 jX_{M2} - \underline{I}_3 jX_L - \underline{I}_2 jX_{M2} = 0$$

$$\underline{I}_1 = \underline{I}_2 + \underline{I}_3 \qquad \Rightarrow I_3 = I_1 - I_2$$

$$X_M = k \cdot \sqrt{X_{L1} X_{L2}} \Rightarrow X_{M1} = 0{,}5\, X_L$$

$$X_{M2} = 0{,}2\, X_L$$

$$E - \underline{I}_1 jX_L - \underline{I}_2 j0{,}5X_L - \underline{I}_2 jX_L - \underline{I}_1 j0{,}5X_L - \underline{I}_3 j0{,}2X_L = 0$$

$$\underline{I}_2 jX_L + \underline{I}_1 j\,0{,}5X_L + \underline{I}_3 j0{,}2X_L - \underline{I}_3 jX_L - \underline{I}_2 j0{,}2X_L = 0$$

$$E - \underline{I}_1 j1{,}5X_L - \underline{I}_2 j1{,}5X_L - \underline{I}_3 j0{,}2X_L = 0$$

$$\underline{I}_2 j0{,}8X_L + j\underline{I}_1 0{,}5X_L - \underline{I}_3 j0{,}8X_L = 0$$

$$E - \underline{I}_1 j1{,}5X_L - \underline{I}_2 j1{,}5X_L - \underline{I}_1 j0{,}2X_L + \underline{I}_2 j0{,}2X_L = 0$$

$$\underline{I}_2 j0{,}8X_L + \underline{I}_1 j0{,}5X_L - \underline{I}_1 j0{,}8X_L + \underline{I}_2 j0{,}8X_L = 0$$

$$E - \underline{I}_1 j1{,}7X_L - \underline{I}_2 j1{,}3X_L = 0$$

$$-\underline{I}_1 j0{,}3X_L + \underline{I}_2 j1{,}6X_L = 0$$

$$\underline{I}_1 = \frac{1{,}6\,X_L}{0{,}3\,X_L}\underline{I}_2 \Rightarrow \underline{I}_1 = \frac{1{,}6}{0{,}3}\underline{I}_2$$

$$\underline{E} - \frac{1{,}6}{0{,}3}\underline{I}_2 j1{,}7X_L - \underline{I}_2 j1{,}3X_L = 0$$

$$\underline{E} - \underline{I}_2 j\left(\frac{1{,}6 \cdot 1{,}7}{0{,}3}X_L + 1{,}3X_L\right) = 0$$

$$\underline{I}_2 = -j\underline{E}\,\frac{30}{311X_L} \Rightarrow \underline{I}_2 = -j\underline{E}\,\frac{30}{311X_L}$$

$$\underline{I}_1 = -j\underline{E}\,\frac{160}{311X_L}$$

$$\underline{I}_3 = -j\underline{E}\,\frac{130}{311X_L}$$

4

1

2

2

2

$$\underline{U}_{L2} = \underline{U}_{S2} + \underline{U}_{M24} + \underline{U}_{M23}$$

$$\underline{U}_{L2} = jX_L \cdot \underline{I}_2 + j0{,}5X_L \cdot \underline{I}_1 + j0{,}2X_L \underline{I}_3 \qquad 2$$

$$\underline{U}_{L2} = jX_L \left(\underline{I}_2 + 0{,}5\,\underline{I}_1 + 0{,}2\,\underline{I}_3 \right)$$

$$= jX_L \left(-j\underline{E}\,\frac{30}{311 X_L} + 0{,}5(-j\underline{E})\frac{160}{311 X_L} + 0{,}2\,(-j\underline{E})\frac{130}{311 X_L} \right)$$

$$\underline{U}_{L2} = j \cdot (-j) \cdot \underline{E} \cdot \frac{136}{311}$$

$$\underline{U}_{L2} = 0{,}4373\,\underline{E}$$

$$\hat{E} = 141{,}42\,V \Rightarrow \underline{E} = \frac{\hat{E}}{\sqrt{2}} e^{j0^\circ} = 100\,e^{j0^\circ}\,V \qquad 2$$

$$\underline{U}_{L2} = 43{,}73\,e^{j0^\circ}\,V$$

Z 15

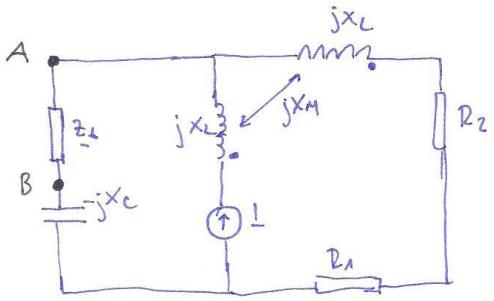
ke 3.

$X_C = X_L = R_2 = R_1 = 5\,\Omega$

$X_M = 2{,}5\,\Omega$

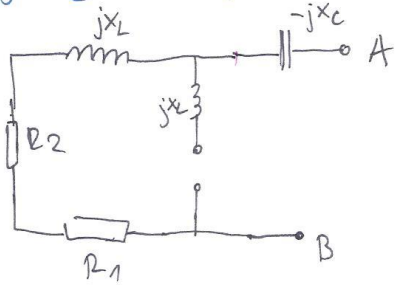
$I = 2\,A$

$\underline{Z}_t = ?$ $P = P_{max}$.



Theveninov teorem!

Određivanje $\underline{Z}_{th}$ → Strujne izvore odpojimo; Naponske kratko spojimo



2

Međuinduktivna veza = 0

Nema struje !!!

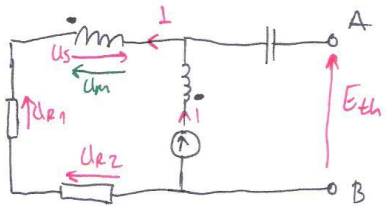
$\underline{Z}_{th} = R_1 + R_2 + jX_L - jX_C =$

$= 10 + j0\ \Omega$

2

$\underline{Z}_{th} = 10\,\Omega$

Određivanje $\underline{E}_{th}$ → Napon priključnica A i B !



2

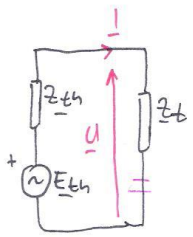
$\underline{E}_{th} = \underline{I}\cdot(R_1+R_2) + j\underline{I}X_L - j\underline{I}X_M$; $\underline{I} = Ie^{j0} = 2\,A$ 2

$\underline{E}_{th} = 2\cdot(10) + j2\cdot 5 - j2\cdot 2{,}5$

$\underline{E}_{th} = 20 + j5$

$\underline{E}_{th} = 20 + j5 = 20{,}62\,e^{j0{,}08\pi}\ V$ 1

Nadomjesna mreža:



1

teorem max. snage:

$\underline{Z}_{th} = \underline{Z}_t^*$ $\quad \underline{Z}_t = \underline{Z}_{th}^*$

za $P = P_{max}$

2

$\underline{Z}_{th} = 10\,\Omega \Rightarrow \underline{Z}_t = 10\,\Omega$

$$\underline{I} = \frac{\underline{E}_{th}}{\underline{Z}_{th} + \underline{Z}_t} = \frac{20{,}62\,e^{j0{,}08\pi}}{20} = 1{,}031\,e^{j0{,}08\pi}$$

1 → štos u izračunu

$$P_{max} = I^2 \cdot Z_t = 1{,}031^2 \cdot 10 = 10{,}63\ W$$

2

$P_{max} = 10{,}63\ W$

Z 15

zadatak 4.

trofazni sustav

Izvor → Y; simetričan

Trošilo → D

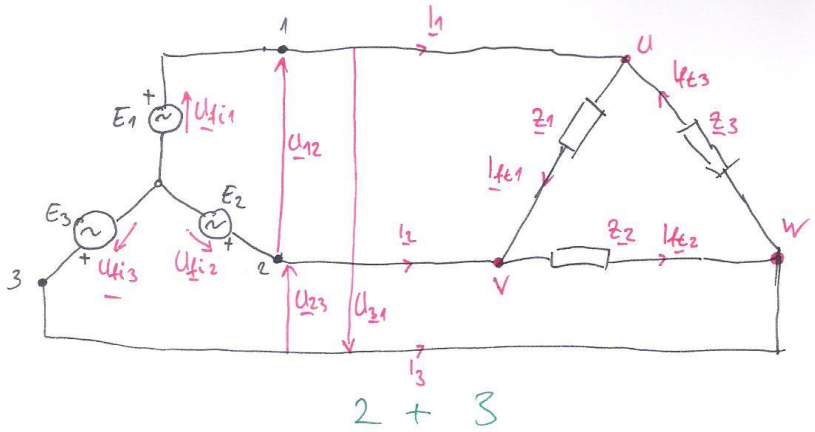
$U_{fi} = 200\,V$

$\underline{Z}_1 = 5 + j5\ \Omega$

$\underline{Z}_2 = 5 + j10\ \Omega$

$\underline{Z}_3 = 5 - j10\ \Omega$

$\underline{S} = ?$ Shema ?



2 + 3

$$\underline{S} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 \qquad 2$$

Linijski naponi referentni!

$$U_L = \sqrt{3}\,U_{fi} = 200\sqrt{3} = 346{,}41\,V$$

$$\underline{U}_{12} = 346{,}41\,e^{j0}\ V$$

$$\underline{U}_{23} = 346{,}41\,e^{-j\frac{2\pi}{3}}\ V$$

$$\underline{U}_{31} = 346{,}41\,e^{+j\frac{2\pi}{3}}\ V$$

2

Izvod za snage:

$$\underline{S}_1 = \underline{U}_{12} \cdot \underline{I}_{fz1}^{*} = \underline{U}_{12} \cdot \frac{\underline{U}_{12}^{*}}{\underline{Z}_1^{*}} = \frac{U_{12}^2}{\underline{Z}_1^{*}}$$

$$\underline{S}_2 = \underline{U}_{23} \cdot \underline{I}_{fz2}^{*} = \underline{U}_{23} \cdot \frac{\underline{U}_{23}^{*}}{\underline{Z}_2^{*}} = \frac{U_{23}^2}{\underline{Z}_2^{*}} \qquad 3$$

$$\underline{S}_3 = \underline{U}_{31} \cdot \underline{I}_{fz3}^{*} = \underline{U}_{31} \cdot \frac{\underline{U}_{31}^{*}}{\underline{Z}_3^{*}} = \frac{U_{31}^2}{\underline{Z}_3^{*}}$$

Iznosi $U_f = U_{12} = U_{23} = U_{31}$

$$\underline{S} = U_f^2 \cdot \left(\frac{1}{\underline{Z}_1^{*}} + \frac{1}{\underline{Z}_2^{*}} + \frac{1}{\underline{Z}_3^{*}} \right) =$$

$$= U_f^2 \left(\frac{1}{5 - j5} + \frac{1}{5 - j10} + \frac{1}{5 + j10} \right)$$

$$\underline{S} = \cancel{X} \cdot U_l^2 \cdot \left(\frac{\sqrt{2}}{10} e^{+j\pi/4} + \frac{\sqrt{5}}{25} e^{+j0{,}35\pi} + \frac{\sqrt{5}}{25} e^{-j0{,}35\pi} \right)$$

$$\underline{S} = \cancel{X} \cdot U_l^2 \left(\frac{\sqrt{2}}{10}\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{10}\frac{\sqrt{2}}{2} + 0{,}04 + \cancel{j0{,}08} + 0{,}04 - \cancel{j0{,}08} \right)$$

$$\underline{S} = \cancel{X} \cdot U_l^2 \left(0{,}1 + 0{,}04 + 0{,}04 + j0{,}1 \right)$$

$\underline{S} = \cancel{64{,}799 + j36\ kVA} = 21{,}6 + j12\ kVA$ 3 Z 15

Zadatak 5.

Trofazni sustav s nulvodičem.

$\underline{Z}_1 = 10\ \Omega$

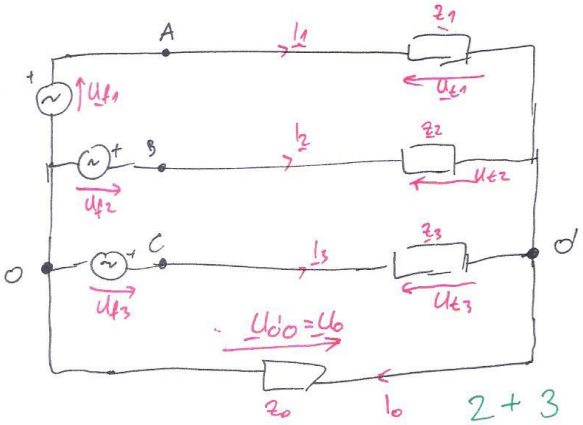
$\underline{Z}_2 = 10 e^{-j30^\circ}\ \Omega$

$\underline{Z}_3 = 10 e^{j30^\circ}\ \Omega$

$U_l = 250\,V \Rightarrow U_f = 144{,}34\,V$

$\underline{Z}_0 = 1\ \Omega$

$\underline{S}_t = ?$ Shema?



2 + 3

$$\underline{S}_t = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 = I_1^2 \cdot \underline{Z}_1 + I_2^2 \cdot \underline{Z}_2 + I_3^2 \cdot \underline{Z}_3$$

Milmanovim teoremom se odreduje U_{oo}:

$$\underline{U}_{oo} = \frac{\frac{\underline{U}_{f1}}{\underline{Z}_1} + \frac{\underline{U}_{f2}}{\underline{Z}_2} + \frac{\underline{U}_{f3}}{\underline{Z}_3}}{\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3} + \frac{1}{\underline{Z}_0}}$$

$$\underline{U}_{oo} = \frac{\frac{\underline{U}_{f1}}{10} + \frac{\underline{U}_{f2}}{10 e^{-j\pi/6}} + \frac{\underline{U}_{f3}}{10 e^{j\pi/6}}}{\frac{1}{10} + \frac{1}{10 e^{-j\pi/6}} + \frac{1}{10 e^{+j\pi/6}} + 1}$$

3

$$\underline{U}_{o'o} = \frac{14,43 + 14,43\,e^{j(\frac{4\pi}{3}+\pi)} + 14,43\,e^{j(\frac{\pi}{3}-\pi/6)}}{0,1 + 2\cdot 0,087 + 1}$$

$$\underline{U}_{o'o} = \frac{14,43 + 14,43\,e^{-j\frac{\pi}{2}} + 14,43\,e^{j\pi/2}}{1,274}$$

$\underline{U}_{o'o} = 11,33\ V$ $\quad \underline{U}_{o'o} = 11,33\ V$ 2

Sada se odredjuje struja trošila:

$\underline{U}_{f1} - \underline{I}_1 \underline{Z}_1 - \underline{U}_{o'o} = 0$ 1

$144,34 - 10\,\underline{I}_1 - 11,33 = 0$

$\underline{I}_1 = 13,301\ A$ 1

$\underline{U}_{f2} - \underline{I}_2 \underline{Z}_2 - \underline{U}_{o'o} = 0$

$144,34\,e^{-j2\pi/3} - \underline{I}_2\,10\,e^{-j\pi/6} - 11,33 = 0$

$-83,5 - j125 - 10\,\underline{I}_2\,e^{-j\pi/6} = 0$

$\underline{I}_2 = 15,03\,e^{j1,64}\ A$

2 = 1 + 1

ili U_f

$\underline{U}_{f3} - \underline{U}_{o'o} = \underline{I}_3 \underline{Z}_3$

$144,34\,e^{+j2\pi/3} - 11,33 = \underline{I}_3\,10\,e^{j\pi/6}$

$-83,5 + j125 = \underline{I}_3\,10\,e^{j\pi/6}$

$\underline{I}_3 = 15,03\,e^{j0,35\pi}\ A$

$\underline{I}_3 = 15,03\,e^{j0,35\pi}\ A$ $\quad 2 = 1+1$

$\underline{U}_{10}\,\underline{I}_1^* = \underline{I}_1 \cdot \underline{Z}_1 \cdot \underline{I}_1^* = I_1^2 \cdot \underline{Z}_1$

Sada se odredjuje trofazna snaga:

$\underline{S}_t = I_1^2 \cdot \underline{Z}_1 + I_2^2 \cdot \underline{Z}_2 + I_3^2 \cdot \underline{Z}_3$ ili $\frac{U^2}{Z}$ 2

$= 13.301^2 \cdot 10 + 15.03^2 \cdot 10\,e^{-j\pi/6} + 15.03^2 \cdot 10\,e^{+j\pi/6}$

$= 13.301^2 \cdot 10 + 2 \cdot (15.03^2 \cdot 10 \cdot \cos(\pi/6))$

$= 5681,88\,e^{j0}\ VA$ 2

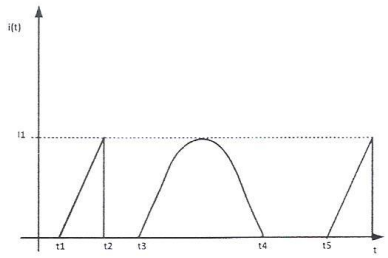
$\Sigma 20$

Ogledni i primjerak

Pismeni ispit 24.06.2016. Ime i prezime:

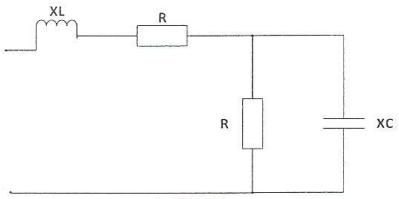
OET 2, preddiplomski studij elektrotehnike

1.ZADATAK;; Kroz otpornik otpora R protječe struja valnog oblika prikazanog na slici 1 (sastavljen od trokutastog i sinusnog valnog oblika). Odredite kolika se količina topline razvije na otporniku u periodu veremena od 0 s do 12 s. Zadano: R= 5 Ohm, t1= 1 s, t2= 2 s, t3= 3 s, t4= 7 s, t5= 9 s, I1=2 A. **(15 bodova).**



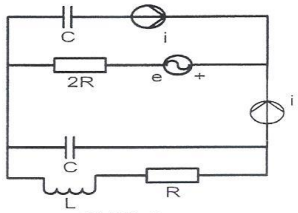
Slika 1.

2.ZADATAK: Električni krug na slici 2 je u rezonanciji. Odredite XC ako je zadano R= 10 Ohm i faktor dobrote kruga Q ~~=3~~ **(20 bodova)** Q=0,3



Slika 2.

3.ZADATAK: Odredite kompleksnu snagu naponskog izvora u nadomijesnoj shemi na slici 3 primjenom metode po izboru. Poznato je: e = (200/√2)*sin(500*t+π/6) V, i = √2*sin(500*t-π/4) A, R = 10 Ω, C= 10 μF i L = 500 mH. Da li izvor troši ili daje djelatnu snagu u mrežu? **(20 bodova)**

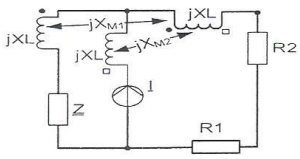


SLIKA 3

4.ZADATAK: Izračunajte najveću djelatnu snagu koja se može razviti na impedanciji Z u nadomijesnoj shemi na slici 4. Zadano je: XL=R2=R1=10 Ω, ~~XM=5~~ Ω, I = 2 A. **(25 bodova)**

X_{M1}= 5 Ω

X_{M2}=3 Ω



SLIKA 4

5.ZADATAK: Trofazno je trošilo spojeno u zvijezdu s neutralnim vodičem. Impedancije faza trošila su: Z1=20∠0°Ω , Z2=10∠-60°Ω i Z3=10∠60°Ω, a linijski napon trofaznog sustava je U = 400 V te je impedancija neutralnog vodiča Z0= 1 ∠0°Ω. Impedancije faznih vodiča trofaznog sustava su Zv= 1+2j Ω. Odredite kompleksnu snagu trošila. Na temelju teksta zadatka nacrtajte odgovarajuću nadomijsenu shemu. **(20 bodova)**

Zadatak:

$t_w = 12s$

$R = 5\Omega$

$t_1 = 1s$

$t_2 = 2s$

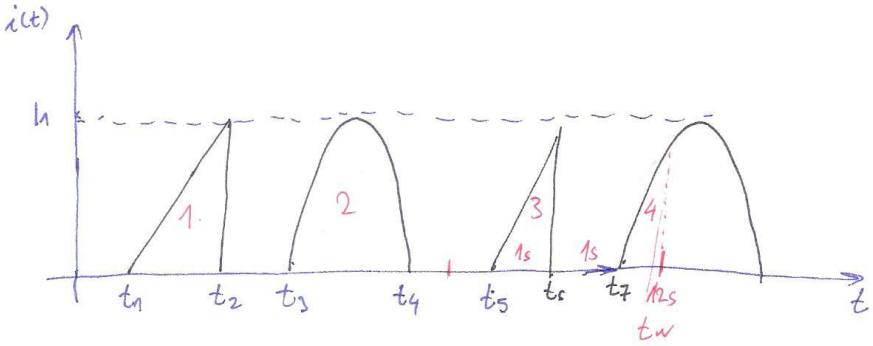
$t_3 = 3s$

$t_4 = 7s$

$t_5 = 9s$

$I_1 = 2A$

$W = ?$



- određuje se količina energije u periodu vremenu koji nije cjelobrojni višekratnik periode strujnog signala. Potrebno je ili: a) integrirati svaki dio
b) odrediti ef. vrijednosti za dio 1-3 + integrirati 4.

10 bodova ako je riješeno na $I_{ef} = \sqrt{\text{zbroj}} \Rightarrow W = I_{ef}^2 \cdot R \cdot t$

Toplina zbog Δ struje:

$$I_{ef\Delta} = \frac{I_1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \Rightarrow W_\Delta = I_{ef\Delta}^2 \cdot R \cdot (t_2 - t_1) + I_{ef\Delta}^2 \cdot R \cdot \underbrace{(t_6 - t_5)}_{t_2 - t_1 \text{ - periodično}}$$ 5

$$W_\Delta = \frac{4}{3} \cdot 5 \cdot 2 \cdot (2-1) = 13{,}333\ J$$

Toplina zbog cijele sinusoide:

$$I_{efn} = \frac{I_1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \Rightarrow W_n = I_{efn}^2 \cdot R \cdot (t_4 - t_3) = 2 \cdot 5 \cdot (7-3) = 40\ J$$ 5

Toplina zbog posljednjeg dijela sinusoide:

$$W_5 = \int_0^{12-t_4} i^2(t) \cdot R\, dt = \int_0^1 I_1 \sin^2(\omega t) \cdot R\, dt = \Rightarrow f = \frac{1}{T} \Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2 \cdot (t_4 - t_3)} = \frac{\pi}{t_4 - t_3} = \frac{\pi}{4}$$

$$= R I_1 \int_0^1 \sin^2\left(\frac{\pi}{4} t\right) dt = R I_1 \int_0^1 \frac{1 - \cos^{\frac{\pi}{2}}\left(\frac{\pi}{2} t\right)}{2} dt = I_1 R \int_0^1 \frac{1}{2} dt - I_1 R \frac{1}{2} \int_0^1 \cos\left(\frac{\pi}{2} t\right) dt =$$ 5

$$= \frac{I_1 R}{2} t \Big|_0^1 - \frac{I_1 R}{\pi} \sin\left(\frac{\pi}{2} t\right) \Big|_0^1 = \frac{I_1 R}{2} - \frac{I_1 R}{\pi} = \frac{5}{2} - \frac{5}{\pi} = 0{,}8085\ J = \frac{10}{2} - \frac{10}{\pi} = 1{,}82\ J$$ 5

Ukupno: $W = W_\Delta + W_n + W_5 = 13{,}33 + 40 + 1{,}82 = 55{,}15\ J$ /20

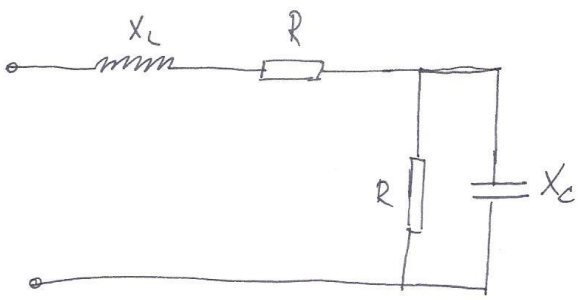
...atak.

$R = 10\,\Omega$

$X_c = ?$

$Q = 0{,}3$

Rezonancija



Rezonancija:

$\mathrm{Re}\{\underline{z}\} = z$

$\mathrm{Im}\{\underline{z}\} = 0$

Paralela:

$$\underline{z}_p = \frac{R\cdot(-jX_c)}{R-jX_c}\cdot\frac{R+jX_c}{R+jX_c} = \frac{-jR^2X_c + RX_c^2}{R^2+X_c^2}$$

$$\underline{z}_p = \frac{RX_c^2}{R^2+X_c^2} - j\frac{R^2X_c}{R^2+X_c^2}$$

$$\underline{z}_{uk} = R + jX_L + \underline{z}_p = R + jX_L + \frac{RX_c^2}{R^2+X_c^2} - j\frac{R^2X_c}{R^2+X_c^2}$$

$$\underline{z}_{uk} = R + \frac{RX_c^2}{R^2+X_c^2} + j\left(X_L - \frac{R^2X_c}{R^2+X_c^2}\right)$$

; Serijska rezonancija izmedu X_L i $\mathrm{Im}\{\underline{z}_p\}$

$\mathrm{Re}\{\underline{z}\}$ $\mathrm{Im}\{\underline{z}\}$ 5

Faktor dobrote:

1. Rezonancija
2. Omjer $U_L/U_R \Rightarrow \frac{X_L}{R}$ — Samo jedan od 2 reaktivna elementa za Ukupni $\underline{z}$ — Ukupni otpor kruga.

$$Q = \frac{X_L}{\mathrm{Re}\{\underline{z}\}} = \frac{X_L}{R + \frac{RX_c^2}{R^2+X_c^2}}$$

$$X_L = Q\cdot R + Q\cdot\frac{RX_c^2}{R^2+X_c^2}$$ 5

Rezonancija:

$$\mathrm{Im}\{\underline{Z}\} = 0$$

$$Q \cdot R + Q \cdot \frac{R X_c^2}{R^2 + X_c^2} = \frac{R^2 X_c}{R^2 + X_c^2} \quad / \cdot R^2 + X_c^2$$

~~$Q R^3 + Q R X_c^2 + Q \cdot R^3 X_c^2 + Q \cdot R X_c^4 =$~~

$$Q R^3 + Q R X_c^2 + Q R X_c^2 - R^2 X_c = 0$$

$$2 Q R X_c^2 - R^2 X_c + Q R^3 = 0$$

$$2 \cdot 0{,}3 \cdot 10 \cdot X_c^2 - 10^2 X_c + 0{,}3 \cdot 10^3 = 0$$

$$6 X_c^2 - 100 X_c + 300 = 0$$

$$X_{c1} = 12{,}74\,\Omega$$

$$X_{c2} = 3{,}92\,\Omega$$

$X_{c1} = 12{,}74\,\Omega$

$X_{c2} = 3{,}92\,\Omega$

5

Σ 15

Zadatak 3.

$e = \frac{200}{\sqrt{2}} \sin(500t + \pi/6)\ V$

$i = \sqrt{2} \sin(500t - \pi/4)\ A$

$R = 10\,\Omega$

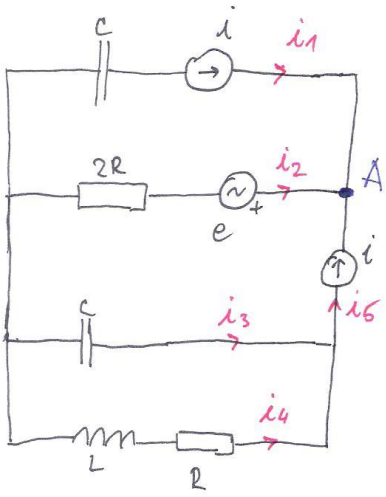
$C = 10\,\mu F$

$L = 500\,mH$

$S_e = ?$

Izvor ili trošilo?

Metoda po izboru



KZS A: $i_1 + i_2 + i_5 = 0$

$i_2 = -i_1 - i_5$

$\underline{I}_2 = -\underline{I}_1 - \underline{I}_5$

$\underline{I}_2 = -1e^{-j\pi/4} - 1e^{-j\pi/4}$

$\underline{I}_2 = -2e^{-j\pi/4}$

$\underline{I}_1 = \frac{\sqrt{2}}{\sqrt{2}} e^{-j\pi/4}\ A$

$\underline{I}_5 = \frac{\sqrt{2}}{\sqrt{2}} e^{-j\pi/4}\ A$

$\underline{E} = \frac{200}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} e^{j\pi/6}$

$\underline{E} = 100 e^{j\pi/6}\ V$

Snaga naponskog izvora:

$\underline{S}_e = \underline{E} \cdot \underline{I}_2^* = 100 e^{j\pi/6} \cdot (-2) e^{j\pi/4}$

$\underline{S}_e = -200 e^{j\frac{5\pi}{12}} = -51{,}76 - 193{,}18\ VA$

$P_e = -51{,}76\ W \Rightarrow$ Izvor radi kao trošilo!!!

ovisno o metodi $\Sigma 20$

Zadatak 4:

$R_1 = R_2 = X_L = 10\,\Omega$

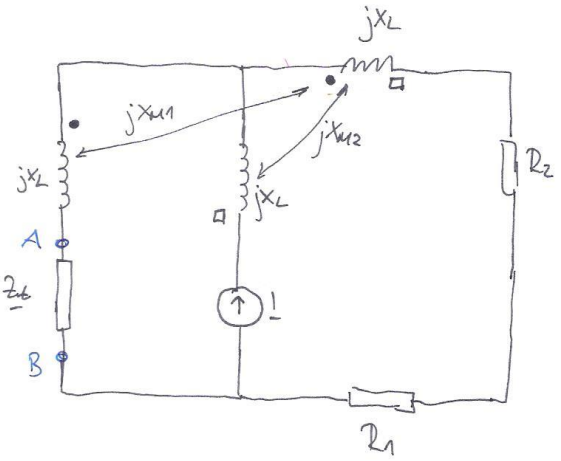
$X_{M1} = 5\,\Omega$; $X_{M2} = 3\,\Omega$

$\underline{I} = 2\,A$

Z_t za $P_{max} = ?$

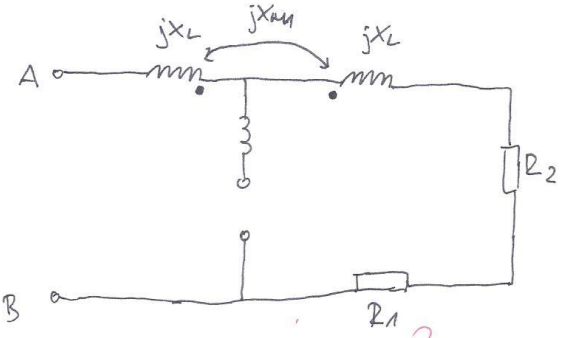
$P_{max} = ?$

$\underline{I} = 2e^{j0}\,A$



Theveninov nadomjesni spoj:

Određivanje $\underline{Z}_{Th}$:



$$\underline{Z}_{TH} = R_1 + R_2 + j(2X_L - 2X_{M1})$$

2

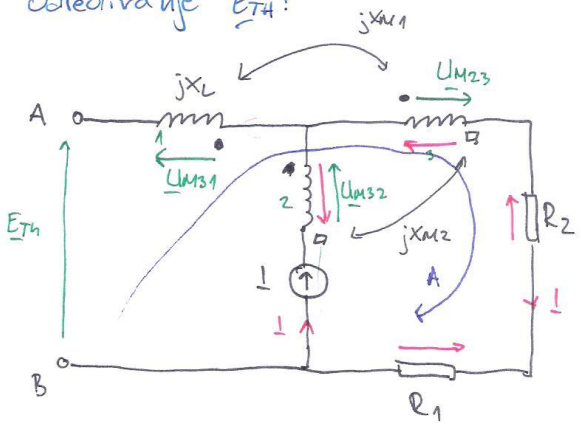
$$\underline{Z}_{TH} = 20 + j10\ \Omega$$

$\underline{Z}_{TH} = \underline{Z}_{TH} = 20 + j10 =$

$= 10\sqrt{5}\,e^{j0{,}14\pi}\ \Omega$

2 3

Određivanje E_{TH}:



2

A: $\underline{E}_{Th} - \underline{U}_{M31} - \underline{U}_{32} + \underline{U}_{M23} - \underline{I}\cdot(R_1 + R_2) = 0$

$$\underline{E}_{Th} = +\underline{I}\cdot jX_{M1} + \underline{I}jX_L - \underline{I}\cdot jX_{M2} + \underline{I}(R_1 + R_2)$$

2

$$\underline{E}_{Th} = 2\cdot(10+10) + j2(5+10-3)$$

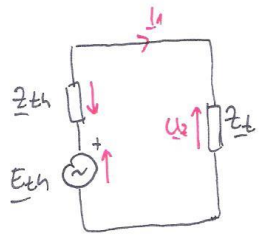
$$\underline{E}_{Th} = 40 + j24\,V$$

$\underline{E}_{Th} = 40 + j24\,V$

$\underline{E}_{Th} = 46{,}65\,e^{j0{,}47}\,V$

2

Odredivanje P_{max}:



Uvjet za P_{max}:

$\underline{Z}_t = \underline{Z}_{th}^*$ 2

$\underline{Z}_t = 20 - j10\,\Omega = 10\sqrt{5}\, e^{-j0,14\pi}\ \Omega$

$\underline{I}_1 = \dfrac{\underline{E}_{th}}{\underline{Z}_{th} + \underline{Z}_t} = \dfrac{46,65\, e^{j0,17\pi}}{40} = 1,17\,A\, e^{j0,17\pi}$

$\cdot 2$

$I_1 = 1,17\, e^{j0,17\pi}\ A$

$\underline{S}_2 = \underline{U}_2 \cdot \underline{I}_1^* = \underline{U}_2 \cdot \underline{I}_1^* = \underline{I}_1 \cdot \underline{Z}_t \cdot \underline{I}_1^* = I_1^2 \cdot \underline{Z}_t$

$\underline{S}_2 = 1,17^2 \cdot 10\sqrt{5}\, e^{-j0,14\pi}$

$\underline{S}_2 = 30,41\, e^{-j0,14\pi}\ VA = 27,52 - j12,95\ VA$

$P_{max} = 27,52\ W$ 3

I20

Zadatak 5.

Trofazni sustav + nul vodič

$\underline{Z}_1 = 20\,e^{j0}\ \Omega$

$\underline{Z}_2 = 10\,e^{-j60°}\ \Omega$

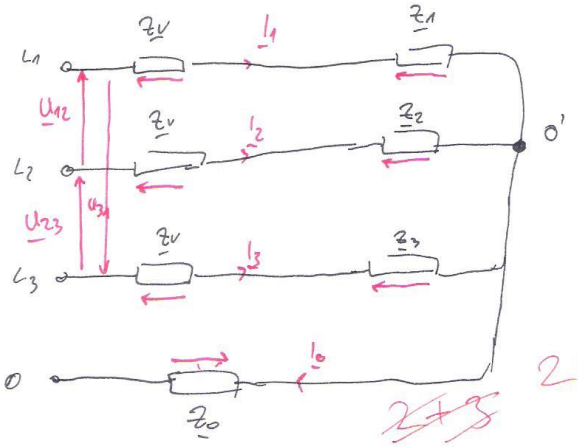
$\underline{Z}_3 = 10\,e^{j60°}\ \Omega$

$U_e = 400\ V$

$\underline{Z}_0 = 1\,e^{j0°}\ \Omega$

$\underline{Z}_v = 1 + j2\ \Omega$

$\underline{S}_t = ?$



2+2

~~2+3~~

1) fazne impedancije:

$\underline{Z}_1' = \underline{Z}_v + \underline{Z}_1 = 1 + j2 + 20 = 21 + j2 = 21{,}095\,e^{j0{,}03\pi}\ \Omega$

$\underline{Z}_2' = \underline{Z}_v + \underline{Z}_2 = 1 + j2 + 10e^{-j60} = 1 + j2 + 5 - j5\sqrt{3}$

$= 6 - j6{,}66 = 8{,}96\,e^{-j0{,}27\pi}\ \Omega$

$\underline{Z}_3' = \underline{Z}_v + \underline{Z}_3 = 1 + j2 + 10\,e^{j60} = 1 + j2 + 5 + j\sqrt{3}\cdot 5$

$= 1 + j2 + 5 + j5\sqrt{3} = 6 + j10{,}66$

$= 6 + j10{,}66 = 12{,}23\,e^{j0{,}33\pi}\ \Omega$

$\underline{Z}_1' = 21 + j2 = 21{,}095\,e^{j0{,}03\pi}\ \Omega$

$\underline{Z}_2' = 6 - j6{,}66 = 8{,}96\,e^{-j0{,}27\pi}\ \Omega$

$\underline{Z}_3' = 6 + j10{,}66 = 12{,}23\,e^{j0{,}33\pi}\ \Omega$

⇓

4

Izvor:

$\underline{U}_1 = 230\,e^{j0}\ V$

$\underline{U}_2 = 230\,e^{-j\frac{2\pi}{3}}\ V$

$\underline{U}_3 = 230\,e^{+j\frac{2\pi}{3}}\ V$

3

225

Millmanov teorem:

$$\underline{U}_{o'o} = \frac{\frac{\underline{U}_1}{\underline{Z}_1'} + \frac{\underline{U}_2}{\underline{Z}_2'} + \frac{\underline{U}_3}{\underline{Z}_3'}}{\frac{1}{\underline{Z}_1'} + \frac{1}{\underline{Z}_2'} + \frac{1}{\underline{Z}_3'} + \frac{1}{\underline{Z}_0}} \quad 3$$

$$\underline{U}_{o'o} = \frac{\frac{230}{21{,}095\, e^{j0{,}03\pi}} + \frac{230\, e^{-j\frac{2\pi}{3}}}{8{,}96\, e^{-j0{,}27\pi}} + \frac{230\, e^{+j\frac{2\pi}{3}}}{12{,}23\, e^{+j0{,}33\pi}}}{\frac{1}{21{,}095\, e^{j0{,}03\pi}} + \frac{1}{8{,}96\, e^{-j0{,}27\pi}} + \frac{1}{12{,}23\, e^{j0{,}33\pi}} + \frac{1}{1}}$$

$$\underline{U}_{o'o} = \frac{10{,}95\, e^{-j0{,}03\pi} + 25{,}67\, e^{-j0{,}4\pi} + 18{,}81\, e^{j0{,}34\pi}}{0{,}047\, e^{-j0{,}03\pi} + 0{,}11\, e^{+j0{,}27\pi} + 0{,}082\, e^{-j0{,}33\pi} + 1}$$

$$\underline{U}_{o'o} = \frac{10{,}30 + 7{,}93 + 9{,}061 + j(-1{,}03 - 24{,}41 + 16{,}48)}{1 + 0{,}0468 + 0{,}073 + 0{,}042 + j(-0{,}0044 + 0{,}023 - j0{,}071)}$$

$$\underline{U}_{o'o} = \frac{27{,}891 - j8{,}96}{1{,}162 + j0{,}0164} = \frac{29{,}29\, e^{-j0{,}1\pi}}{0{,}163\, e^{j0{,}032\pi}} = \frac{29{,}29\, e^{-j0{,}1\pi}}{1{,}162\, e^{+j0{,}005\pi}}$$

$\underline{U}_{o'o} = 179{,}69\, e^{-j0{,}13\pi}$ V

$\underline{U}_{o'o} = 25{,}21\, e^{-j0{,}11\pi}$ V

$\underline{U}_{o'o} = 25{,}21\, e^{-j0{,}11\pi}$ V 2

$= 25{,}07 - j8{,}54$

Struje trošila:

$$\underline{I}_1 = \frac{\underline{U}_1 - \underline{U}_{10}}{\underline{Z}_1'} = \frac{230 - 25{,}07 + j8{,}54}{21.095\, e^{j0.03\pi}} = \frac{205{,}11\, e^{-j0{,}013\pi}}{21.095\, e^{j0.03\pi}} = 9{,}72\, e^{-j0{,}043\pi}\ A$$

2

$$\underline{I}_2 = \frac{\underline{U}_2 - \underline{U}_{10}}{\underline{Z}_2'} = \frac{230\, e^{-j\frac{2\pi}{3}} - 25{,}07 + j8{,}54}{8{,}96\, e^{-j0.27\pi}} = \frac{-115 - j200 - 25{,}07 + j8{,}54}{8{,}96\, e^{-j0.27\pi}} =$$

$$= \frac{-140{,}07 - j191{,}46}{8{,}96\, e^{-j0.27\pi}} = \frac{237{,}23\, e^{+j1{,}8\pi}}{8{,}96\, e^{-j0.27\pi}} = 26{,}48\, e^{j1{,}57\pi}$$

2

$$\underline{I}_3 = \frac{\underline{U}_3 - \underline{U}_{10}}{\underline{Z}_3} = \frac{230\, e^{+j\frac{2\pi}{3}} - 25{,}07 + j8{,}54}{12{,}23\, e^{j0{,}33\pi}} = \frac{-115 + j200 - 25{,}07 + j8{,}54}{12{,}23\, e^{j0{,}33\pi}}$$

$$\underline{I}_3 = \frac{-140{,}07 + j208{,}54}{12{,}23\, e^{j0.33\pi}} = \frac{251{,}21\, e^{j0{,}69\pi}}{12{,}23\, e^{j0.33\pi}} = 20{,}54\, e^{j0{,}35\pi}\ A$$

2

Kompleksna snaga trošila:

$$\underline{S} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 =$$

2

$$= I_1^2 \cdot \underline{Z}_1 + I_2^2 \cdot \underline{Z}_2 + I_3^2 \cdot \underline{Z}_3 =$$

$$= 9{,}72^2 \cdot 20 + 26{,}48^2 \cdot 10\, e^{-j\pi/3} + 20{,}54^2 \cdot 10\, e^{+j\pi/3} =$$

$$= 1889{,}57 + 7011{,}9\, e^{-j\pi/3} + 4218{,}92\, e^{j\pi/3} =$$

$$= 1889{,}57 + 3505{,}95 + 2103{,}46 + j(-6072{,}48 + 3653{,}69)$$

$$= 7504{,}98 - j2418{,}78 = 7885{,}13\, e^{-j0{,}1\pi}\ VA$$

1

Osnove elektrotehnike 2, PISMENI ISPIT

8. srpanj 2016.

Ime: .....
Prezime: .....
Broj Indexa: .....

Broj bodova: ...../100

Zadatak 1 Jednofazni transformator nominalne snage od 100 kVA ima stupanj korisnog djelovanja $\eta = 95\%$ uz nominalnu snagu i faktor snage trošila jednak 1. Omjer gubitaka u bakru i gubitaka u željezu je 1.5 a omjer napona gornjonaponske i donjonaponske strane u praznom hodu je $\frac{10000}{500}$. Odredite otpore gornjonaponskog i donjonaponskog namota ako su gubici u bakru u oba namota jednaki.

Zadatak 2 Zadan je krug na slici 1a gdje je $u = \frac{90}{\sqrt{2}}\sin(314t)$. Otpor je 10Ω,a kapacitivna reaktancija je 20Ω. Odredite iznos induktiviteta L ako je efektivna vrijednost struje izvora $i = 3.5\sin(314t - 0.27\pi)$A.

Zadatak 3 Zadana je trofazna mreža (slika 1b) s linijskim naponom efektivne vrijednosti 220V. Odredite snagu koju uzima trošilo te nacrtajte vektoroski dijagram napona i struja ako su $R = X_c = X_l = 100\Omega$.

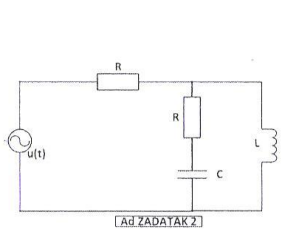
Zadatak 4 Spoj sa slike 2a je u rezonanciji. Odredite L i C ako je rezonantna frekvencija $5 * 10^4$ Hz i djelatna snaga izvora 100 W. Pri tome struje iznose: $I_1 = 3A$, $I_2 = 2A$. $I = 7.24A$. $R_1 = R_3$.

Zadatak 5 Odredite koeficijent međuindukcije za krug prema slici 2b pri kojemu će nastati strujna rezonancija. Induktiviteti svitaka su: $L_1 = 40$ mH, $L_2 = 20$ mH. Kružna frekvencija izvora je $\omega = 10^4\ s^{-1}$. Kapacitet kondenzatora je $C = 0.125\mu$F.

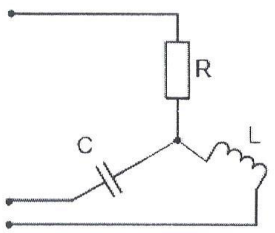
Bodovanje zadataka

Z1	Z2	Z3	Z4	Z5	Σ
20	20	20	20	20	100

Sl. 1

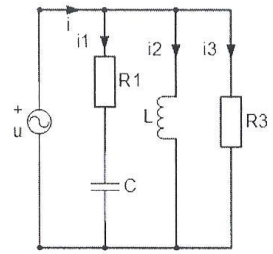


(a)

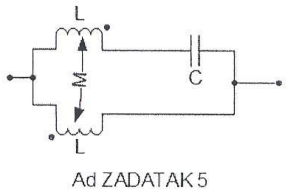


(b)

Sl. 2



(a)



(b)

Ogledni primjerak.

Osnove elektrotehnike 2

Pismeni ispit - 8.7.2016.

- R J E Š E NJ A -

Zadatak 1.

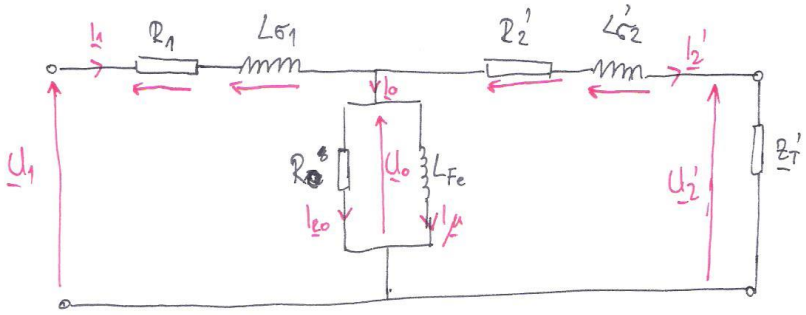
1-fazni transformator

100 kVA $\eta = 95\%$ uz $\cos\varphi_2 = 1$ i $S = S_n$

$P_{Cu}/P_{Fe} = 1{,}5$

$N_1/N_2 = U_{1o}/U_{2o} = 10000/500 \Rightarrow n = 20$

$R_1 = ?$; $R_2 = ?$; $P_{Cu1} = P_{Cu2}$



Iz zadanih podataka:

$$\frac{P_1}{P_2} = 0{,}95$$

$$P_2 = 100\ \text{kW} \quad (\cos\varphi_2 = 1) \Rightarrow P_1 = \frac{100 \cdot 10^3}{0{,}95} = 105{,}26\ \text{kW}$$

$$U_2 = U_{2n} = 500\ \text{V} \Rightarrow U_2' = n \cdot U_2 = 20 \cdot 500 = 10\ \text{kV}$$

$$P_{Cu1} = P_{Cu2} = 1{,}5\, P_{Fe}$$

$P_1 = P_2 + 2 \cdot P_{cu} + P_{Fe}$

$P_1 = P_2 + 2P_{cu} + \frac{P_{cu}}{1,5}$

$P_{cu} = \frac{P_1 - P_2}{1,333} = \frac{(10526 - 100) \cdot 10^3}{1,333} = 3,95\ kW$

$P_{cu1} = P_{cu2} = 3,95\ kW$ 5

Teorijska napomena:

Gubici u bakru realnog transformatora određuju se iz pokusa kratkog spoja.

Tada se poprečna grana zanemaruje i struje I_1 i I_2' postaju jednake!

$I_2' = \frac{U_2'}{Z_2'} = \frac{S_2}{U_2'} = \frac{100 \cdot 10^3}{10 \cdot 10^3} = 10\ A$ 5

Pretpostavka vezana za P_{cu}:

$I_1 = I_2'$

$P_{cu1} = I_1^2 \cdot R_1 \Rightarrow R_1 = \frac{P_{cu1}}{I_1^2} = \frac{3,95 \cdot 10^3}{10^2} = 39,5\ \Omega$

$P_{cu2} = I_2'^2 \cdot R_2' \Rightarrow R_2' = \frac{P_{cu2}}{I_2'^2} = 39,5\ \Omega$

$R_2' = n^2 \cdot R_2 \Rightarrow R_2 = \frac{R_2'}{n^2} = \frac{39,5}{20^2} = 0,1\ \Omega$ 5

Zadatak 2:

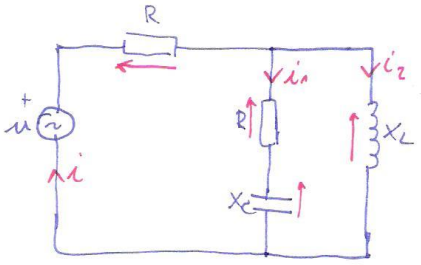
$u = \frac{90}{\sqrt{2}} \sin(314t)$ [V]

$i = 3{,}5 \sin(314t - 0{,}27\pi)$ [A]

$R = 10\,\Omega$

$X_c = 20\,\Omega$

$L = ?$



$$\underline{U} = \frac{90/\sqrt{2}}{\sqrt{2}} e^{j0} \text{ [V]} = 45 e^{j0} \text{ V}$$

$$\underline{I} = \frac{3{,}5}{\sqrt{2}} e^{-j0{,}27\pi} = 2{,}47 e^{-j0{,}27\pi} \text{ A}$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{45 e^{j0}}{2{,}47 e^{-j0{,}27\pi}} = 18{,}18 e^{j0{,}27\pi} = 12{,}02 + j13{,}64 \text{ [}\Omega\text{]}$$

5

$$\underline{Z} = R + \underline{Z}_{par} = R + \frac{jX_L \cdot (R - jX_c)}{R + j(X_L - X_c)} = \frac{R - j(X_L - X_c)}{R - j(X_L - X_c)}$$

$$= R + \frac{jX_L R (R - jX_c) - j^2 X_L (R - jX_c)(X_L - X_c)}{R^2 + (X_L - X_c)^2} =$$

$$= R + \frac{jX_L R^2 - j^2 X_L R X_c - j^2 X_L R (X_L - X_c) + j^3 X_L X_c (X_L - X_c)}{R^2 + (X_L - X_c)^2} =$$

$$= R + \frac{jX_L R^2 + X_L R X_c + X_L R (X_L - X_c) - jX_L X_c (X_L - X_c)}{R^2 + (X_L - X_c)^2}$$

5

$\text{Re}\{\underline{Z}\} + j\,\text{Im}\{\underline{Z}\}$

$$\mathrm{Re}\{\underline{Z}\} = 12{,}02 = R + \frac{R X_L X_C + R(X_L^2 - X_L X_C)}{R^2 + (X_L - X_C)^2} \Big/ \cdot\, R^2 + (X_L - X_C)^2$$ 5

$$12{,}02 R^2 + 12{,}02 (X_L - X_C)^2 = R^3 + R(X_L - X_C)^2 + R X_L X_C + R(X_L^2 - X_L X_C) \Big/ : R$$

$$12{,}02 R + \frac{12{,}02}{R}\left[X_L^2 - 2X_L X_C + X_C^2\right] = R^2 + \left[X_L^2 - 2X_L X_C + X_C^2\right] + X_L X_C + X_L^2 - X_L X_C$$

$$12{,}02 R + \frac{12{,}02}{R} X_L^2 - 2 \cdot \frac{12{,}02}{R} X_C X_L + \frac{12{,}02}{R} X_C^2 = \ldots$$

$$\ldots = R^2 + X_L^2 - 2X_C X_L + X_C^2 + X_L X_C + X_L^2 - X_L X_C$$

$$\left(\frac{12{,}02}{R} - 1 - 1\right) X_L^2 + \left(-2\frac{12{,}02}{R} X_C + 2X_C\right) X_L + \left(12{,}02 R + \frac{12{,}02}{R} X_C^2 - R^2 - X_C^2\right) = 0$$

$$-0{,}798 X_L^2 - 8{,}08 X_L + 101 = 0$$

$X_{L1} = 7{,}27\ \Omega$ $\quad X_{L1} = 7{,}27\ \Omega$

~~$X_{L2} = -17{,}4\ \Omega$~~

$$X_L = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{7{,}27}{314} = 23{,}2\ \mathrm{mH}$$ 5

$L = 23{,}2\ \mathrm{mH}$

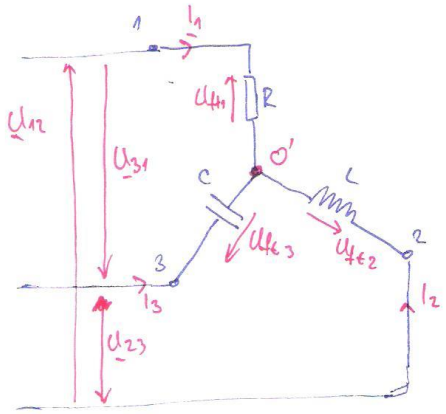
Zadatak 3:

$U_e = 220\ V$

$R = X_C = X_L = 100\ \Omega$

$S_T = ?$

Fazorski dijagram



$U_e = 220V \Rightarrow U_f = \frac{U_l}{\sqrt{3}} = 127{,}02V \quad U_f = 127{,}02V$

$\underline{U}_{f1} = 127{,}02\, e^{j0}\ V$

$\underline{U}_{f2} = 127{,}02\, e^{-j\frac{2\pi}{3}}\ V$

$\underline{U}_{f3} = 127{,}02\, e^{+j\frac{2\pi}{3}}\ V$

$G = B_L = B_C = \frac{1}{100} = 0{,}01\ S$

Milman:

$$\underline{U}_{oo'} = \frac{\underline{U}_1\underline{Y}_1 + \underline{U}_2\underline{Y}_2 + \underline{U}_3\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} \qquad -jB_L$$

$$\underline{U}_{oo'} = \frac{127{,}02\, e^{j0}\cdot 0{,}01 + 127{,}02\, e^{-j\frac{2\pi}{3}}\cdot(-j0{,}01) + 127{,}02\, e^{+j\frac{2\pi}{3}}\cdot(+j0{,}01)}{0{,}01 - j0{,}01 + j0{,}01}$$

$$\underline{U}_{oo'} = \frac{1{,}27 + 1{,}27\, e^{-j\frac{7\pi}{6}} + 1{,}27\, e^{+j\frac{7\pi}{6}}}{0{,}01}$$

$$\underline{U}_{oo'} = \frac{1{,}27 + (-1{,}099 + j0{,}635) + (-1{,}099 - j0{,}635)}{0{,}01}$$

$\underline{U}_{oo'} = -92{,}8\ V \qquad U_{oo'} = -92{,}8\ V$

Fazni naponi trošila:

$$\underline{U}_{ft1} = \underline{U}_{f1} - \underline{U}_{do} = 127{,}02\, e^{j0} + 92{,}8 = 220\,V \qquad U_{ft1} = 220V$$

$$\underline{U}_{ft2} = \underline{U}_{f2} - \underline{U}_{do} = 127{,}02\, e^{-j\frac{2\pi}{3}} + 92{,}8 = -63{,}51 - j110 + 92{,}8$$

$$U_{ft2} = 29{,}29 - j110\ V = 113{,}84\, e^{-j0{,}42\pi}\ [V]$$

$$\underline{U}_{ft3} = \underline{U}_{f3} - \underline{U}_{do} = 127{,}02\, e^{+j\frac{2\pi}{3}} + 92{,}8 = -63{,}51 + j110 + 92{,}8 =$$

$$U_{ft3} = 29{,}29 + j110V = 113{,}84\, e^{+j0{,}42\pi}\ [V]$$

Fazne (linijske) struje: 5

$$\underline{I}_1 = \frac{\underline{U}_{ft1}}{R} = \frac{220}{100} = 2{,}2\, e^{j0}\ A$$

$$\underline{I}_2 = \frac{\underline{U}_{ft2}}{jX_L} = \frac{113{,}84\, e^{-j0{,}42\pi}}{100\, e^{+j\pi/2}} = 1{,}13\, e^{-j0{,}92\pi}\ A$$

w Dobro bilo ... $1{,}13\, e^{-j0{,}92\pi}$ A

$$\underline{I}_3 = \frac{U_{ft3}}{-jX_C} = \frac{113{,}84\, e^{+j0{,}42\pi}}{100\, e^{-j\pi/2}} = 1{,}13\, e^{+j0{,}92\pi}\ A$$

Snaga trošila:

$$\underline{S} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 = I_1^2 \cdot R + I_2^2 \cdot jX_L - I_3^2 \cdot jX_C =$$

$$= 2{,}2^2 \cdot 100 + \left(1{,}13\, e^{-j0{,}92\pi}\right)^2 \cdot j100 + \left(1{,}13\, e^{+j0{,}92\pi}\right)^2 \cdot (-j100) =$$

$$= 2{,}2^2 \cdot 100 + 127{,}69\, e^{j0,}$$

Snaga trošila:

$$\underline{S} = \underline{U} \cdot \underline{I}^* = \underline{I} \cdot \underline{Z} \cdot \underline{I}^* = \underline{I} \cdot \underline{I}^* \cdot \underline{Z} = I^2 \cdot \underline{Z}$$

$$\underline{S} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 =$$

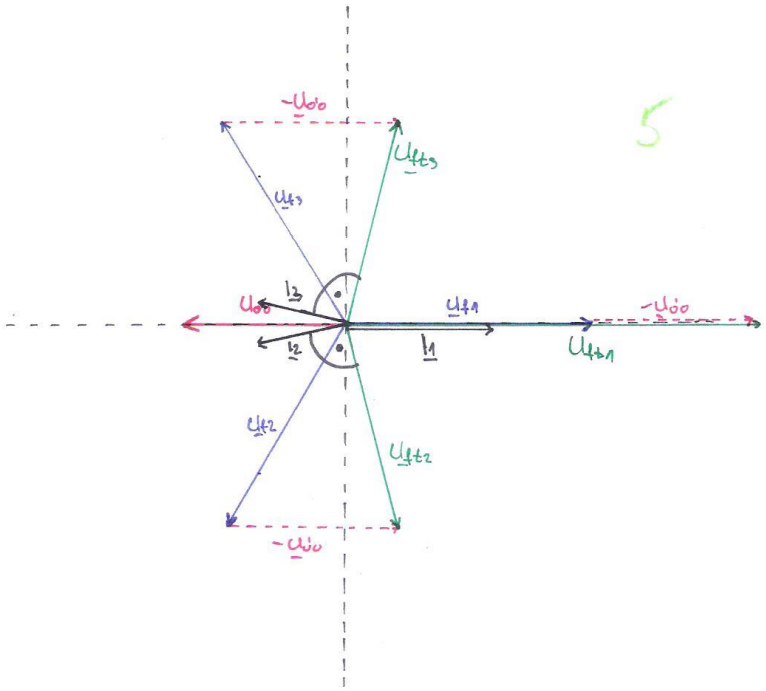
$$= I_1^2 \cdot \underline{Z}_1 + I_2^2 \cdot \underline{Z}_2 + I_3^2 \cdot \underline{Z}_3 =$$

$$= 2{,}2^2 \cdot 100 + 1{,}13^2 \cdot j100 - 1{,}13^2 \cdot j100 =$$

$$\underline{S} = 484\, e^{j0}\ \text{VA} = P$$

5

Fazorski dijagram:



5

Zadatak 4:

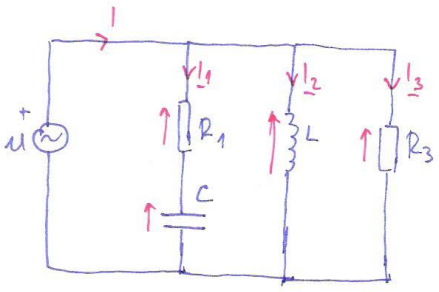
$f = 5\,kHz$ Rezonancija

$I_1 = 3\,A$; $I_2 = 2\,A$; $I = 7{,}24\,A$

$R_1 = R_3$

$P = 100\,W$

$L; C = ?$



Neka je:

$$\underline{U} = U e^{j0^\circ}\ [V]$$

Tada je zbog rezonancije i:

$$\underline{I} = I e^{j0}\ [A] = 7{,}24 + j0\ A$$

KZS:

$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$\underline{I} = I_{1Re} + jI_{1im} - jI_2 + I_3$$

(čisti L: $jI_{1im} - jI_2$; čisti R: $I_{1Re} + I_3$)

$$I_{1Re} + I_3 = 7{,}24$$

$$I_{1im} - I_2 = 0$$

$$I_{1im} = I_2 = 2\,A$$

$$I_1 = \sqrt{I_{1Re}^2 + I_{1im}^2} \Rightarrow$$

$$I_{1Re} = \sqrt{I_1^2 - I_{1im}^2} = \sqrt{3^2 - 2^2} = \sqrt{5}$$

$$I_3 = 7{,}24 - I_{1Re} = 7{,}24 - \sqrt{5}$$

$$I_3 = 5\,A$$

Iz snage se određuje R:

$$P = I_1^2 \cdot R + I_3^2 \cdot R$$

$$R = \frac{P}{I_1^2 + I_3^2} = \frac{100}{3^2 + 5^2}$$

$$R = 2{,}94\,\Omega$$ 5

Sada napon izvora:

$$U = I_3 \cdot R = 5 \cdot 2{,}94 = 14{,}71\,V$$ 5

Sada X_L:

$$X_L = \frac{U}{I_2} = \frac{14{,}71}{2} = 7{,}35\,\Omega$$

5

$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{7{,}35}{2\pi \cdot 5 \cdot 10^3} = 0{,}23\,mH$$

X_c se mora fazorski:

$$\underline{U} = \underline{I}_1 R - jX_c \cdot \underline{I}_1 \qquad ; \quad \underline{I}_1 = \sqrt{5} + j2 \ A$$

$$\frac{\underline{U}}{\underline{I}_1} = R - jX_c$$

$$\mathrm{Im}\left\{\frac{\underline{U}}{\underline{I}_1}\right\} = X_c$$

$$\frac{\underline{U}}{\underline{I}_1} = \frac{14,71}{3e^{j0,23\pi}} = 4,9\, e^{-j0,23\pi} = 3,68 - j3,24\,[\Omega]$$

$$X_c = 3,24\,\Omega$$

$$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_c} = \frac{1}{2\pi f X_c} = \frac{1}{2\pi \cdot 5 \cdot 10^3 \cdot 3,24}$$

$C = 9,82\,\mu F$ 5

Zadatak 5

$L_1 = 40\,mH$ $\quad X_{L1} = \omega L_1 = 400\,\Omega$

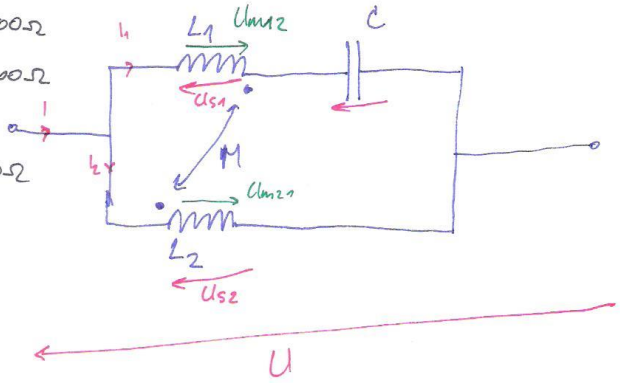
$L_2 = 20\,mH$ $\quad X_{L2} = \omega L_2 = 200\,\Omega$

$\omega = 10^4\,s^{-1}$

$C = 0{,}125\,\mu F$ $\quad X_C = \frac{1}{\omega C} = 800\,\Omega$

$M = ?$

Rezonancija!



Pronaći impedanciju kao omjer U i I $\qquad \underline{Z} = \frac{\underline{U}}{\underline{I}}$

KZN:

$$U_{m12} - U_{s1} - U_c - U_{m21} + U_{s2} = 0$$

$$jX_M \cdot \underline{I}_2 - jX_{L1} \cdot \underline{I}_1 + jX_C \underline{I}_1 - jX_M \cdot \underline{I}_1 + jX_{L2} \cdot \underline{I}_2 = 0$$

$$\underline{I}_2 (jX_M + jX_{L2}) + \underline{I}_1 (-jX_{L1} + jX_C - jX_M) = 0$$

$$\underline{I}_2 (jX_{L2} + jX_M) = \underline{I}_1 (jX_{L1} + jX_M - jX_C)$$

$$\underline{I}_2 = \underline{I}_1 \cdot \frac{jX_{L1} + jX_M - jX_C}{jX_{L2} + jX_M} \qquad (1)$$

$\Rightarrow$ j se može izostaviti

5

KZS: $\quad \underline{I} = \underline{I}_1 + \underline{I}_2$

$$\underline{I} = \underline{I}_1 \left(1 + \frac{jX_{L1} + jX_M - jX_C}{jX_{L2} + jX_M} \right)$$

5

$\Rightarrow$ j se može izostaviti

KZN I OHM:

$$\underline{U} = \underline{I} \cdot \underline{Z} = I_1 \left(1 + \frac{X_{L1} + jX_M - X_C}{X_{L2} + X_M}\right) \cdot \underline{Z}$$

jedn (1)

$$\underline{U} = \underline{I}_2 jX_{L2} - \underline{I}_1 jX_M = I_1 \left(\frac{X_{L1} + X_M - X_C}{X_{L2} + X_M} \cdot jX_2 - jX_M\right)$$

$$\left(1 + \frac{X_{L1} + X_M - X_C}{X_{L2} + X_M}\right)\underline{Z} = \frac{X_{L1} + X_M - X_C}{X_{L2} + X_M} jX_2 - jX_M$$

$$\underline{Z} = j \frac{\frac{(X_{L1} + X_M - X_C) X_2}{X_{L2} + X_M} - X_M}{1 + \frac{X_{L1} + X_M - X_C}{X_{L2} + X_M}}$$

$$\underline{Z} = j \frac{(X_{L1} + X_M - X_C) X_2 - X_M X_{L2} - X_M^2}{X_{L2} + X_M + X_{L1} + X_M - X_C}$$

$\underline{Z} = 0$; Rezonancija: $\Rightarrow$ Brojnik $= 0$

$$(X_{L1} + X_M - X_C) X_2 - X_M X_{L2} - X_M^2 = 0$$

$$-X_M^2 + X_M (X_{L2} - X_{L2}) + X_{L2} X_{L1} - X_{L2} X_C = 0$$

$$-X_M^2 = X_{L2} X_C - X_{L2} X_{L1}$$

$$X_M = \sqrt{-X_{L2} X_C + X_{L2} X_{L1}}$$

$$X_M = \sqrt{-200 \cdot 800 + 200 \cdot 400}$$

$X_M = \ldots$ kompleksno rješenje! Nema koeficijenta međuinduktivnosti za koji se postiže rezonancija!